COMPUTE WHERE IT COUNTS: EFFICIENT LARGE LANGUAGE MODELS VIA LEARNED GRANULAR SPARSITY

Cyris Kissane*, Niveditha Iyer*, & Adam Klein*
Crystal AI
{cyris,niveditha,adam}@crystalai.org

ABSTRACT

Sparsity-aware inference can dramatically shrink computation requirements by reducing the number of parameters, and thus FLOPs, used in each forward pass. Existing methods tend to be heuristic (zeroing activations below fixed thresholds, retaining top K activations, etc.). These methods do not directly optimize individual thresholds using gradient-based methods and experience sharp performance degradation beyond 50% sparsity. This paper describes CWIC (Compute Where it Counts), a method that makes sparsity thresholds learnable and contextual. CWIC is designed to enable conditional computation in models, allowing them to self-distribute sparsity across each weight matrix. In addition, it enables models to allocate different amounts of compute for each token and sequence. We also propose "granular sparsity" that decomposes matrix columns into smaller "stripes" for more expressive sparsity patterns. We show that CWIC and granular sparsity can distill 2x to 6x compute-efficient sparse models from Llama 3.2-1B and 3B. We find that sparsifying Llama3.2-1B to 66% sparsity (3x active parameter reduction) with CWIC achieves a 15% increase in aggregate benchmark scores over doing the same with TEAL Liu et al. (2025) while speeding up inference wall clock times by 2.5x in both GPU and CPU settings. CWIC shows promising scaling behavior. A CWIC Llama3.2-3B at 66% sparsity outperforms Llama3.2-1B (equal number of active parameters) on standard benchmarks by 1.6%, with wall clock improvements of 2.7x in both GPU and CPU settings. Notably, CWIC models are found to allocate little compute to filler or replicated text and more to challenging benchmark questions.

1 INTRODUCTION

As large language models (LLMs) grow in parameter count to attain desired performance levels, their inference compute requirements grow in lockstep. Consumer devices cannot support large inference compute needs which now drive massive industry hardware expenses. This poses a bottleneck for many applications, especially agentic ones, that require high-speed, low-cost options for real-world deployment. Several methods have been proposed to improve LLM inference efficiency, including sparse Mixture of Experts (MoE) (Shazeer et al., 2017), quantization (Jin et al., 2024), ReLU-based sparsity (Mirzadeh et al., 2024), and activation sparsity (Lee et al., 2024; Liu et al., 2025; Zhang et al., 2025).

Quantization casts weights into lower-precision types. This tends to be less expressive than sparsity techniques as the same matrix is applied to all inputs. ReLU-based sparsity requires that the model use ReLU activations; however, modern models have adopted SwiGLU (Shazeer, 2020a) and other alternatives for superior performance. Activation sparsity methods such as CATS (Lee et al., 2024) zero out non-salient, activations lower than a threshold. Although sparsity can be input-dependent, no current activation sparsity method directly learns activation thresholds. Furthermore, these prior works exhibit sharp performance degradation at sparsity exceeding 50% .

*All authors contributed equally.

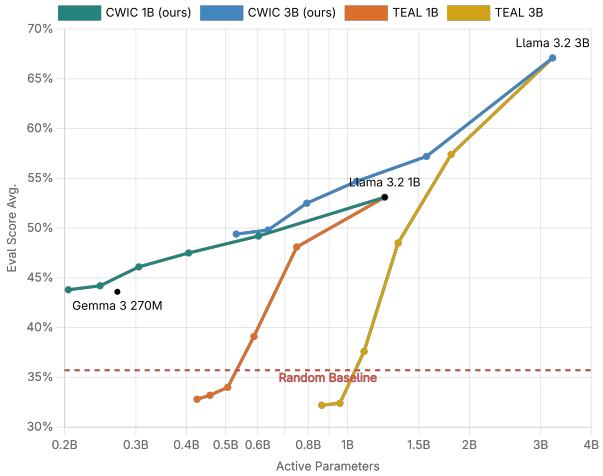

Figure 1: Average performance of CWIC and active-parameter equivalent models on standard LLM benchmarks (described in subsection 4.2). Each trend line shows performance of a base model at varying levels of sparsity (or active parameters) under CWIC and TEAL. The rightmost point of each trendline corresponds to 0% sparsity.

This work presents CWIC (Compute Where it Counts), a method to effectively train sparsity-aware models. CWIC is inspired by sparse autoencoders (Rajamanoharan et al., 2024) and uses straight-through-estimator (Bengio et al., 2013) to directly optimize activation thresholds (Lee et al., 2024).

This allows the model to (1) designate different levels of sparsity to different weight matrices and (2) dynamically allocate compute to different tokens and sequences. It also gives us control over the desired sparsity level using a loss function.

To complement CWIC, we introduce *granular sparsity*, which partitions matrix columns into smaller *stripes*. This enables increased expressivity over the traditional approach of treating a matrix column as the "unit" of sparsity that can be turned "on" or "off" (Lee et al., 2024). Picking a stripe size of 512 retains the hardware acceleration associated with these methods.

We distill 1B and 3B models from the Llama 3.2 family (Grattafiori et al., 2024) under the CWIC framework into versions that use 2x-6x fewer Active Parameters (AP) per token. As in , we find that CWIC outperforms TEAL (Liu et al., 2025) and exhibits a graceful performance tradeoff at higher sparsity levels (lower AP) while other activation sparsity techniques tend to exhibit performance collapse beyond 33% sparsity (3x reduction in AP). CWIC kernels match TEAL performance and CWIC scales as a sparsification method.

Examining the FLOPs and Active Parameters assigned by CWIC models to benchmark tasks reveals that they naturally allocate less compute to "easier" tokens (such as role tokens, filler words, system prompt) and sequences (such as questions from ARC-Easy vs ARC-Challenge (Clark et al., 2018)).

2 Related Work

Activation sparsity reduces computation requirements by zeroing small activations, allowing them to be skipped during matrix multiplications. Relufication (Mirzadeh et al., 2024) replaces pretrained LLM activation functions with ReLUs and inserts ReLUs elsewhere in the model to induce sparsity. After finetuning to recover performance, Relufication can reduce FLOP counts by up to 50% with almost no degradation. ProSparse (Song et al., 2025) builds on Relufication by adding an L1 penalty to ReLU activations to further increase sparsity.

GRIFFIN, Dong et al. (2024) exploits sequence-level activation similarity to define adaptive sequence-level sparsity patterns. Deja Vu (Liu et al., 2023) and ShadowLLM (Akhauri et al., 2024) predict sparsity on the fly by training small auxiliary MLPs to determine which weights matter to particular input sequences. Q-Sparse (Wang et al., 2024) discards all but the K largest channels of in-

put vectors when computing linear layers. Q-sparse improves performance over compute-equivalent dense models and shows that sparsity degrades performance less on larger models.

Most similar to our work are CATS (Lee et al., 2024), TEAL (Liu et al., 2025), and R-SPARSE (Zhang et al., 2025), which zero all activations that are smaller than a *threshold*. However, unlike our work, they do not directly optimize individual thresholds using gradient-based methods. CATS targets the same activation frequency with every threshold, TEAL optimizes thresholds using a greedy block-wise heuristic, and R-SPARSE uses a search algorithm in conjunction with singular value analysis.

Mixture of Experts (MoE) activates certain sections of the neural network ("experts") to reduce active parameter counts. Unlike activation sparsity, MoE architectures typically use a learned routing mechanism to choose which experts to activate. Sparsely-Gated Mixture-of-Experts (Shazeer et al., 2017) proposed a gating network that incentivizes sparse, yet balanced, expert selection for language modeling and translation. DeepSeekMoE (Dai et al., 2024) demonstrated that combinatorial expert selection with more experts improves performance. Pham et al. (2024) demonstrate the selection of experts with the largest output magnitude is an effective routing strategy. Zhou et al. (2022) find that performance can improve if different tokens can receive different amounts of compute.

Sparse Autoencoders (SAEs) can faithfully reconstruct the hidden state of a neural network while activating a very small percentage of their features. Variants include top-k SAEs (Gao et al., 2024) that retain only the k largest activations, and JumpReLU (Rajamanoharan et al., 2024) SAEs that retain only those ReLU activation that exceed a learned threshold. Notably, JumpReLU makes sparsity learnable, and allows different numbers of features to activate for different examples. Ayonrinde (2024) showed reconstruction fidelity is improved when different numbers of features can be activated for different tokens.

3 METHODS

3.1 GRANULAR SPARSITY

Existing sparsity methods (Mirzadeh et al., 2024; Wang et al., 2024; Lee et al., 2024) exploit column sparsity (sometimes transposed and described as row sparsity) in matrix multiplications. When an input vector has a zero element, the computation for the corresponding matrix column can be skipped. Thus columns, which are either entirely used or entirely unused, and can be considered the units of conditional computation.

Inspired by the insight of DeepSeekMoE (Dai et al., 2024) that smaller and more configurable experts lead to better performance, we sought to create a more expressive sparsity mechanism. Termed "granular sparsity", our method breaks each column into a set of *stripes* - with each stripe being activated individually. This greatly increases the number of achievable sparsity configurations (see Figure 2). We construct the Granular Matrix Multiplication (GMM) operation to multiply only those stripes of the weight matrix that are kept "active". The GMM algorithm is explained below.

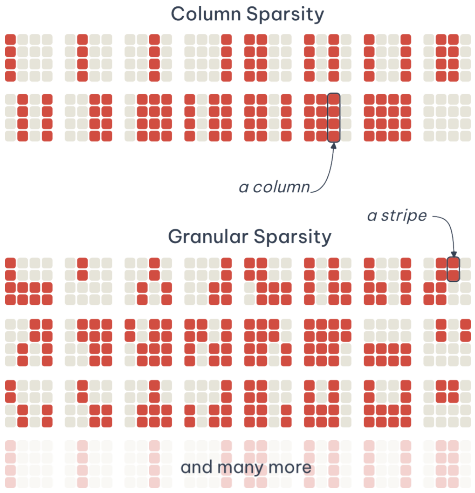

Figure 2: Given a 4x4 matrix, using column sparsity results in 16 possible configurations. Partitioning each column into 2 individually activated stripes results in a significantly greater number of achievable configurations.

The multiplication of a matrix $\boldsymbol{W} \in \mathbb{R}^{m \times n}$ with a vector $\boldsymbol{x} \in \mathbb{R}^n$, can be expressed as the sum of $\boldsymbol{W}$ column vectors $\boldsymbol{v}_i := \boldsymbol{W}_{:,i}$ weighted by the elements x_i of $\boldsymbol{x}$.

$$\boldsymbol{y} := \boldsymbol{W}\boldsymbol{x} = \sum_{i=1}^{n} x_i \boldsymbol{v}_i$$

Column-wise sparse matrix multiplication, uses a binary mask $\mathcal{M} \in \{0,1\}^n$ to zero out certain elements of $\boldsymbol{x}$. The remaining components contribute to the multiplication.

$$\boldsymbol{y}_{\text{sparse}} := \sum_{i=1}^{n} \mathcal{M}_i x_i \boldsymbol{v}_i = \sum_{i \in S_{\mathcal{M}}} x_i \boldsymbol{v}_i \text{ where } S_{\mathcal{M}} := \{i \in \{1,\dots,n\} \mid \mathcal{M}_i = 1\}$$

In our method, we partition the output vector $\boldsymbol{y}$ and each column vector $\boldsymbol{v}_i$ into k equally sized *stripes*. We introduce $\Big\|$, a concatenation operator where $\boldsymbol{y} = \Big\|_{r=1}^{k} \boldsymbol{y}^{(r)}$.

$$\boldsymbol{y} := \Big\|_{r=1}^{k} \sum_{i=1}^{n} x_i \boldsymbol{v}_i^{(r)}$$

Our granular sparsity operation, denoted $\text{GMM}(\boldsymbol{x}, \boldsymbol{W}, \mathcal{G})$, uses a binary mask $\mathcal{G} \in \{0,1\}^{k \times n}$ that zeroes out certain stripes in the matrix $\boldsymbol{W}$, as shown in Figure 2. We define the set of active (non-zeroed) stripe indices:

$$S_{\mathcal{G}} = \{(r,i) \in \{1,\dots,k\} \times \{1,\dots,n\} \mid \mathcal{G}_{r,i} = 1\}$$

Then the granular sparse output is given by:

$$\begin{aligned} \boldsymbol{y}_{\text{granular}} = \text{GMM}(\boldsymbol{x}, \boldsymbol{W}, \mathcal{G}) &:= \Big\|_{r=1}^{k} \sum_{i=1}^{n} \mathcal{G}_{r,i} x_i \boldsymbol{v}_i^{(r)} \\ &= \Big\|_{r=1}^{k} \sum_{(r,i) \in S_{\mathcal{G}}} x_i \boldsymbol{v}_i^{(r)} \end{aligned}$$

Note that when $k = 1$, granular matrix multiplication reduces to standard column-wise sparsity.

3.2 Sparsity Thresholds

We use a different mask $\mathcal{G}$ for each input vector x (a strategy known as *contextual sparsity*) (Liu et al., 2023). To determine the $\mathcal{G}$, we use the magnitudes of each element in x. Specifically, we learn a grid of thesholds $\theta \in \mathbb{R}_{+}^{k \times n}$ such that $\mathcal{G}_{r,i}$ is 1 if and only if x_i has have a magnitude of at least $\theta_{r,i}$. We define this using a function $T(x,t)$ that given the absolute value of x gate and the threshold returns an one or zero using the Heaviside step function $H(z)$.

Similarily, we denote granular matrix multiplication parameterized by thresholds with $\text{GMM}(x, W, \theta)$.

Calculating $\mathcal{G}$

$$\begin{aligned} H(z) &:= \begin{cases} 1 & z \geq 0 \\ 0 & z < 0 \end{cases} \\ T(x,t) &= H(x,t) \\ \mathcal{G}_{r,i} &= T(|x_i| - \theta_{r,i}) \end{aligned}$$

GMM parameterized by thresholds

$$\begin{aligned} y_{\text{granular}} &= \text{GMM}(x, W; \theta) \\ &:= \Big\|_{r=1}^{k} \sum_{i=1}^{n} H(|x_i| - \theta_{r,i})\ x_i v_i^{(r)} \end{aligned}$$

We initialize θ to zero at the start of training. To keep θ positive, we set $\theta = \max(0, \theta)$ after every parameter update. We found that θ benefited from a significantly higher learning rate (LR) than other parameters. We set the LR of θ to the base LR multiplied by $\eta_\theta \sqrt{n}$, where η_θ is a hyperparameter.

3.3 Learning Thresholds

Previous works that use thresholds to determine contextual sparsity masks often rely on heuristics to determine threshold values (Lee et al., 2024). We seek better optimization by directly learning the thresholds. Unfortunately, $H(z)$ is not differentiable. We thus build on the ideas introduced in JumpReLU (Rajamanoharan et al., 2024) to construct a straight-through-estimator (Bengio et al., 2013) with a pseudo-derivative that approximates the true derivative. This pseudo-derivative is defined as follows, with K representing a kernel function and ϵ representing a tunable bandwidth:

$$\frac{\partial}{\partial z} H(z) := \frac{1}{\epsilon} K\Big(\frac{z}{\epsilon}\Big)$$

For our kernel function K, we use the rectangle function seen in JumpReLU:

$$K(z) := H\big(z - \frac{1}{2}\big) - H\big(z + \frac{1}{2}\big)$$

When calculating either $\frac{\partial}{\partial x_i}\mathcal{G}_{r,i}$ or $\frac{\partial}{\partial \theta_{r,i}}\mathcal{G}_{r,i}$ we set the corresponding ϵ_i equal to the batch-wise standard deviation of x_i (which does not receive gradients), scaled by a constant uniform hyperparameter α_ϵ with $\epsilon_i := \alpha_\epsilon \text{std}(x_i)$. For a more detailed analysis of this gradient estimator, we refer readers to the JumpReLU paper (Rajamanoharan et al., 2024).

When backpropagating the AP loss grads on the AP mask $T(x, t)$ we allow grads to flow to both x, and t. When backpropagating the grads from $\mathcal{G}_{r,i}x_i = T(|x_i| - \theta_{r,i})x_i$ we make two key changes. First, we take x_i' grads straight through as if x_i was unmasked. This was informed by our ablation studies. Second, we stop the gradients from flowing to $|x_i|$. This is needed because the scales of grads going to x_i through $|x_i|$ were dominating in practice due to the normalization step outlined in subsection 3.4.

3.4 NORMALIZATION

When initializing from a pre-trained network, we found that the batch-wise scales and offsets of x_i values can vary throughout the network, making it difficult to tune hyperparameters. To remedy this, we whiten x before it enters the matrix multiplication operation. For a batch of x vectors, we calculate the batch-wise mean $\bar{x} \in \mathbb{R}^n$ and standard deviation $\sigma(x) \in \mathbb{R}^n$.

Then, we then perform the following: $y_{\text{granular}} = \text{GMM}\Big(x - \bar{x}, W; \theta \odot \sigma(x)\Big) + W\bar{x}$

Note that when θ is composed of zeroes, this whitening procedure does not effect y_{sparse}. To increase training stability, we track $\bar{x}$ and $\sigma(x)$ on a rolling basis. For this, the hyperparameter β_{dist} is used to compute an exponential moving average. At inference time the running $\bar{x}$ and $\sigma(x)$ values from the last step of training are used.

3.5 STRAIGHT-THROUGH ESTIMATION

Previous work (Wang et al., 2024) shows that sparse models can benefit from using straight-through estimators during training. In our case, that means taking the gradients of x as if there was no sparsity. Specifically, we use the following definition for the gradients of x: $\nabla_x\ y_{\text{granular}} := W^\top$

Although this modification obviously leads to biased gradients, we theorize that this estimator improves performance by removing the variance imparted on the grads when the values of $\mathcal{G}$ are changing frequently. The STE could also fix the vanishing gradients associated with high sparsity, as postulated by Q-Sparse (Wang et al., 2024). The gradients of θ and W are left unchanged.

3.6 CONTROLLING SPARSITY

A key advantage of the learnable threshold is that we can control the sparsity of the model using a loss function. The number of parameters to compute $\text{GMM}(x, W; \theta)$, is:

$$\text{APs}(x, W, \mathcal{G}) := \frac{m}{k}||\mathcal{G}||_1$$

We define $\text{APs}(B)$ to represent the number of active parameters required by the entire model to operate on a batch B, $\text{APs}_{\text{base}}(B)$ to represent the number of active parameters required if we did not have sparsity, and $\text{APs}_{\text{target}}(B)$ to represent the desired Active Parameter count. We then define the Active Parameter Reduction (APR) as the ratio between the base FLOP count and the sparse FLOP count and our loss function:

$$\text{APR}(B) := \frac{\text{APs}_{\text{base}}(B)}{\text{APs}(B)} \qquad \text{APR}_{\text{target}}(B) := \frac{\text{APs}_{\text{base}}(B)}{\text{APs}_{\text{target}}(B)}$$

$$\mathcal{L}_{\text{APs}} := \Big[\min\Big(\text{APR}(B) - \text{APR}_{\text{target}}(B), 0\Big)\Big]^2$$

This loss was chosen as it gives us control over the desired compute costs, and stable performance during training. We found it important to include a warmup phase where $\text{APs}_{\text{target}}(B)$ is incrementally lowered. We linearly increase $\text{APR}_{\text{target}}(B)$ from 1.05, to our final target value + 0.05.

3.7 Feed-Forward Network Modifications

Previous works have found that intermediate activations of the feed-forward blocks exhibit natural sparsity (Mirzadeh et al., 2024). To leverage this, we slightly modify our granular sparsity system for use in the feed-forward blocks. The Llama 3 suite of models uses gated linear units (GLU) (Shazeer, 2020b) as their feedforward design. These are parameterized by a gate matrix $W_{\text{gate}} \in \mathbb{R}^{n \times d}$, an up matrix $W_{\text{up}} \in \mathbb{R}^{n \times d}$, and a down matrix $W_{\text{down}} \in \mathbb{R}^{d \times n}$.

We compute $W_{\text{gate}}x$ with the standard granular sparsity method. Then, we use our learned threshold method to compute $\mathcal{M} \in \{0,1\}^d$ based on the magnitudes of $\text{silu}(W_{\text{gate}}x)$.

$$a = \mathcal{M} \odot \text{silu}\big(\text{GMM}(x, W_{\text{gate}}, \mathcal{G})\big)$$
$$y_{\text{GLU, granular}} = W_{\text{down}}\Big(W_{\text{up}} \odot a\Big)$$

When the operations rendered unnecessary by the mask are filtered out, we arrive at the FLOP cost of this operation.

$$\text{APs}(\mathcal{G}, \mathcal{M}) = \frac{d}{k}||\mathcal{G}||_1 + 2n||\mathcal{M}||_1$$

3.8 Model Distillation

To efficiently train our sparse models, we use knowledge distillation (Hinton et al., 2015) with a teacher network (usually the one that was used to initialize the sparse model).
For our distillation loss, over a sequence of length T, we use a combination of the forward (FKL) and reverse KL divergence (RKL), which has been shown to work better than either divergence individually (Wu et al., 2024).

$$\text{FKL} := \sum_{t=1}^{T} \text{KL}\Big(p_{\text{t}}(y_t|y_{<t}), p_{\text{s}}(y_t|y_{<t})\Big)$$
$$\text{RKL} := \sum_{t=1}^{T} \text{KL}\Big(p_{\text{s}}(y_t|y_{<t}), p_{\text{t}}(y_t|y_{<t})\Big)$$
$$\mathcal{L}_{\text{distill}} := \frac{1}{2}\Big(\text{FKL} + \text{RKL}\Big)$$

Our total loss is a weighted combination of our distillation loss and our Active Parameter loss described in subsection 3.6:

$$\mathcal{L} = \mathcal{L}_{\text{distill}} + \lambda_{\text{APs}}\mathcal{L}_{\text{APs}}$$

4 Experiments

4.1 Setup

We tested our methods with the Llama-3.2-1B [1] and Llama-3.2-3B [2] models for both the teacher and the initialization of the student. We used the AdamW optimizer. Details of our hyperparameters can be found in Appendix B and details of our training set in Appendix C.

Training was carried out over approximately 1B tokens for both 1B and 3B parameter experiments, with our 1B model requiring 52 hours on 1 H100 GPU (for a total of 52 GPU hours). This compute equates to only 0.015% of the 370K H100 hours originally used to train Llama-3.2-1B [3]. Our training implementation used only Python-level Pytorch operations. We believe that a lower level implementation of the GMM operation, akin to Flash Attention (Dao et al., 2022), could accelerate training considerably.

As a baseline, we applied TEAL (Liu et al., 2025), the current state of the art activation sparsity method, to the base model using default settings. To generate activations for threshold optimization, we used 1000 sequences from our training dataset.

[1] meta-llama/Llama-3.2-1B
[2] meta-llama/Llama-3.2-3B
[3] meta-llama/Llama-3.2-1B

We measure model performance on standard LLM benchmarks. Specificaly, we aggregate performance over MMLU (Hendrycks et al., 2021), WinoGrande (Sakaguchi et al., 2019), ARC (Easy and Challenge) (Clark et al., 2018), HellaSwag (Zellers et al., 2019), PIQA (Bisk et al., 2019) and OpenBookQA (Mihaylov et al., 2018).

4.2 RESULTS

4.2.1 CWIC OUTPERFORMS TEAL

We find that CWIC outperforms TEAL (Liu et al., 2025) across all sparsity levels. CWIC shows a gradual performance tradeoff as sparsity levels are increased (lower AP). This is explained by the increasing KL divergence we observe between high APR models and the original model. In contrast, TEAL tends to exhibit performance collapse beyond 33% sparsity (3x reduction in AP). We find that sparsifying Llama3.2-1B to 66% sparsity with CWIC achieves a 15% increase in aggregate benchmark scores over doing the same with TEAL. This performance gap between TEAL and CWIC grows as the sparsity levels increase. The full set of results can be found in subsection A.1.

4.2.2 CWIC SPARSIFICATION OUTPERFORMS ACTIVE PARAMETER EQUIVALENTS

Table 1 compares our 3x APR models to dense transformer models in their compute classes that have been trained from scratch. We see that they marginally outperform the dense model with matching AP on the benchmark aggregate. In Figure 1, a 3x AP reduction of Llama3.2-3B using TEAL underperforms Llama3.2-1B (an AP equivalent model) by roughly 16%. In contrast, doing the same with CWIC outperforms Llama3.2-1B by 1.6%.

Model Type (Active Params)	**ALL**	**MMLU**	**WG***	**Arc-C**	**HS***	**Arc-E**	**OBQA***	**PIQA**
Llama3B CWIC 3.05x APR (1053M)	54.7	38.3	58.3	39.5	64.3	69.4	39.4	73.9
Llama1B (1236M)	53.1	38.6	57.9	36.9	64.2	61.7	37.2	74.9
Llama1B CWIC 3.05x APR (245M)	44.2	25.2	51.6	29.2	48.3	52.8	33.4	68.8
Gemma-3-270M	43.6	24.3	52.7	27.6	43.8	57.5	30.6	68.9

Table 1: Comparison of 3x APR models to compute-equivalent models. WG, HS and OBQA are the WinoGrande, HellaSwag and OpenbookQA datasets respectively.

4.2.3 CWIC ABLATION FINDINGS

We construct two ablations to test CWIC design choices and run them on Llama-3.2-1B. Our first ablation tests the utility of "granular sparsity" by reverting to column sparsity instead of stripe sparsity. In the second ablation, we including MSE on the last hidden states in the distillation loss. At APR 3.05x on Llama-3.2-1B, we observe a 1.7% performance hit using column sparsity and a 1.6% performance drop with MSE on the last hidden states (see Table 2). The full set of ablation results are provided in Appendix A.2.

Version	**Step**	**Multiplier**	**all**	**mmlu**	**wg**	**arc:c**	**HS**	**arc:e**	**obqa**	**piqa**
Full CWIC	6000	3.05x	47.5	26.7	55.7	33.1	53.6	58.0	34.8	70.4
No Stripes	6000	3.05x	45.8	26.2	54.1	30.9	51.4	56.3	32.6	69.1
No MSE	6000	3.05x	45.9	24.7	52.2	30.7	51.4	57.4	34.4	70.5

Table 2: Model performance under CWIC ablations

4.2.4 CWIC SCALING LAWS

Given budget constraints, we sparsified 1B and 3B models. We observed CWIC versions of both outperform their respective TEAL sparsified and dense active-parameter-equivialents across sparsity levels (full results in

Modern LLMs can have 10-100x more parameters. Q-Sparse (Wang et al., 2024) studied the scaling laws of sparsely activated models and found that the performance gap between dense and sparsely-activated models diminishes as model size increases. TEAL Liu et al. (2025) shows that at 65%

sparsity, Llama-3-8B4 sees an average downstream task performance reduction of 22% while Llama-3-70B5 (70B parameters) only sees a performance reduction of 9%. Since our method and TEAL both calculate sparsity based on activation thresholds, we expect to see similar scaling trends where CWIC will work as well, if not better, at larger scales

4.2.5 Wall Clock Speedups

We implemented a triton kernel based on TEAL (Liu et al., 2025). Keeping the stripe size a multiple of the kernel block size along the output dimension maintains the efficiency of TEAL's triton kernel. In the GPU setting we matched the TEAL procedure. For testing on CPU we implemented a Rust version and compared to OpenBlas, taking precautions to evict the matrix from cache before each timed operation. At 66% sparsity, CWIC kernels speed up inference by 2.5x in both GPU and CPU settings for Llama3.2-1B and 2.7x for LLama3.2-3B.

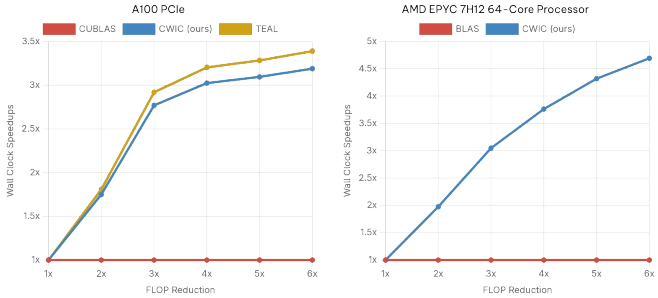

Figure 3: GPU and CPU speedups where hidden width: 4096, stripe size: 512

5 Discussion

5.1 Sparsity Patterns

Analyzing the activation frequencies reveals several insights about the model's structure of circuits and sparsity. First, we observe that some channels in the residual stream activate for almost every input across layers (Figure 7). They emerge very early (100 steps), and stay almost unchanged for the entirety of training. We believe that these channels may capture common knowledge, and serve a similar purpose to the shared experts used by DeepSeekMoE (Dai et al., 2024). An example of this behavior can be found in Figure D.

Second, we find striking patterns (see Figure D) in the O attention matrix. Individual attention heads have consistently high/low activations across channels. We hypothesize that the model is learning to "prune" unhelpful attention heads, similar to previous work that reduces compute cost by explicitly removing attention heads (Mugnaini et al., 2025).

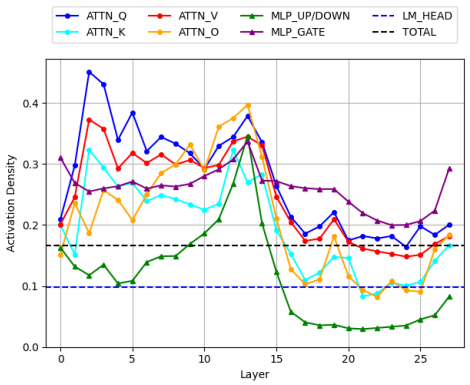

Figure 4: Activation frequency of different matrix types across layers from the 6x APR checkpoint of Llama-3.2-3B. FFN and FN GATE represent the W_{up} / W_{down} matrices and W_{gate} respectively.

Unlike the QKV, UP, GATE layers there are no patterns across layers as the input to O is not the residual stream. We have provided an example of this sparsity pattern in Appendix D.

We also find that stripes in the language modeling head corresponding to earlier vocabulary tokens have higher activation frequencies. This is because the vocabulary of Llama models is implicitly sorted from high to low token frequency, so frequent tokens have more compute designated to them

than infrequent ones. This effect is so pronounced that without regularization, the network will eventually put nearly *zero* compute towards infrequent tokens, leading to representation collapse.

Finally, among the Q, K, and V attention matrices, activation of the V matrix is the most dense, followed by K and O (Figure 4). Note the dip in activation frequencies across matrices from layers 8-14 followed by a slight rise in layers 15 and 16.

5.2 Variable Compute Budgets

Figure 5 uses text thickness to indicate compute allocated to different tokens in a prompt-response pair on a CWIC model based off Llama-3.2-1B-Instruct. We observe three common trends in compute allocation in general.

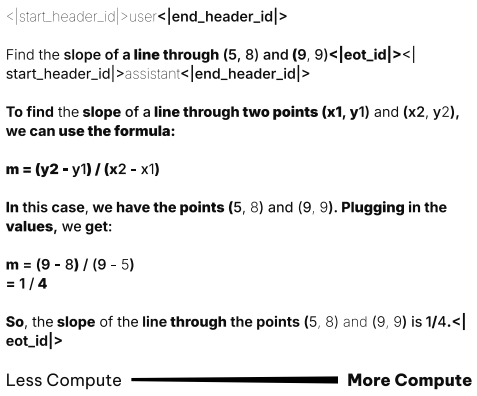

Figure 5: Active parameter count allocation of a 6x APR model across tokens.

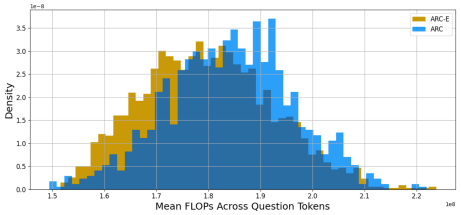

Figure 6: Active parameters allocated to each question of ARC-Easy and ARC-Challenge. We ignore the system prompt and only consider the question and option tokens. Active parameters are averaged per-question.

Semantically quoting sections of the user prompt such as "(5,8) and (9,9)" in the response uses fewer active parameters. Punctuation, prepositions and filler words such as "the" and "and" are allocated lower budgets. Behavior on system prompt and system/user/assistant role tokens gets distilled into very few active parameters and thus receives a very low compute allocation. We provide more visualizations of variable compute allocation in Appendix D.

Sparsity thresholds allows different tokens and different sequences to use different amounts of compute. Figure 6 shows that the average Active parameters dedicated by the 6x APR model to running ARC-Easy and ARC-Challenge benchmarks follow similar distributions, but with the Arc-E taking 5% less compute on average. A significant number of the ARC-E questions were allocated less compute than ARC-C questions. As evidenced by the scores of the 6x APR model (subsection A.1), ARC-Easy questions are indeed easier for the model!

6 Future Work

Our striping method groups output channels based on their order. However, outside of attention heads, there is no guarantee that adjacent channels are functionally similar. When initializing a sparse model from a pretrained one, it may be beneficial to reorder channels to form semantic groupings. This idea has seen success in mixture-of-expert conversions (Elazar & Taylor, 2022).

We observed that the KL divergence between the base and APR models remains stable between 2x-4.5x APR. It curves up beyond 4.5x APR. Performance improvements could be realized by modifying the APR warm-up to use a non-linear schedule.

We believe that several further optimizations can be made to the GPU CWIC kernel to unlock further speed gains from Active Parameter reductions.

Finally, we observed that longer training in tokens improves quality (as seen with the continued 6x training). It is natural to ask how far can this be pushed, and what are the scaling laws.

LIMITATIONS

The primary limitation of our work concerns the scale of our experiments. We did not train models larger than 3B parameters, and we did not train for longer than 1B tokens. We believe that significantly better benchmark performance could be achieved with more training. Previous work (Wang et al., 2024) has also indicated that sparsity leads to less performance degradation at larger scales, so our method may be more suitable for larger models than those tested here.

Furthermore, we only tested our method on transformer architectures and language modeling. The application to other models such as vision transformers (Dosovitskiy et al., 2021) has not been explored.

REPRODUCIBILITY STATEMENT

We are committed to publishing reproducible research. We will open source our code (codebase already ready and version tracked on GitHub) and training data (already uploaded to HuggingFace). We will also be releasing model checkpoints for the research community to use and test out of the box (already uploaded to HuggingFace). We will link these resources in the paper after the review session during which anonymity is required. As a starting measure, we have provided all hyperparameter settings in Appendix B and the training data recipe in Appendix C.

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A Evaluations Results

A.1 Comparison to Compute Equivalent Models

Model	Technique	Step	Multiplier	all	mmlu	wg	arc:c	HS	arc:e	obqa	piqa
Gemma270M	—	—	—	43.6	24.3	52.7	27.6	43.8	57.5	30.6	68.9
llama-3.2-1B	—	—	—	53.1	38.6	57.9	36.9	64.2	61.7	37.2	74.9
llama-3.2-1B	CWIC	3000	2.05x	49.2	27.7	56.0	33.5	58.4	60.2	35.8	72.9
llama-3.2-1B	CWIC	6000	3.05x	47.5	26.7	55.7	33.1	53.6	58.0	34.8	70.4
llama-3.2-1B	CWIC	9000	4.05x	46.1	26.4	53.3	30.2	50.3	56.9	36.2	69.2
llama-3.2-1B	CWIC	12000	5.05x	44.2	25.2	51.6	29.2	48.3	52.8	33.4	68.8
llama-3.2-1B	CWIC	15000	6.05x	43.8	24.9	52.2	30.6	46.6	53.4	30.8	68.1
llama-3.2-1B	CWIC	18000	6.05x	44.5	26.0	53.4	31.0	48.0	53.9	31.4	68.0
llama-3.2-3B	—	—	—	61.7	55.2	65.1	46.3	74.1	72.1	40.8	78.1
llama-3.2-3B	CWIC	3000	2.05x	57.2	47.4	60.1	42.2	68.3	69.1	38.0	75.6
llama-3.2-3B	CWIC	6000	3.05x	54.7	38.3	58.3	39.5	64.3	69.4	39.4	73.9
llama-3.2-3B	CWIC	9000	4.05x	52.5	32.3	57.0	38.6	61.5	67.1	37.0	74.3
llama-3.2-3B	CWIC	12000	5.05x	49.8	28.8	53.6	35.2	59.1	62.6	37.4	71.7
llama-3.2-3B	CWIC	15000	6.05x	49.4	27.6	53.8	34.3	57.7	62.2	38.2	72.0
llama-3.2-1B	TEAL	—	1.65x	48.1	28.1	55.3	34.0	55.2	56.6	35.6	71.7
llama-3.2-1B	TEAL	—	2.11x	39.1	24.1	51.3	24.7	39.0	43.0	30.0	61.9
llama-3.2-1B	TEAL	—	2.44x	34.0	22.9	48.0	22.7	28.9	32.8	28.0	55.0
llama-3.2-1B	TEAL	—	2.70x	33.2	22.9	48.5	24.9	26.3	29.0	28.4	52.0
llama-3.2-1B	TEAL	—	2.91x	32.8	23.0	49.4	25.1	26.0	26.6	29.8	50.1
llama-3.2-3B	TEAL	—	1.78x	57.4	48.3	58.6	42.4	69.8	69.1	38.4	75.6
llama-3.2-3B	TEAL	—	2.41x	48.5	30.7	54.5	35.2	55.4	58.3	36.0	69.4
llama-3.2-3B	TEAL	—	2.92x	37.6	23.2	50.1	26.6	34.5	41.7	26.0	61.3
llama-3.2-3B	TEAL	—	3.35x	32.4	23.0	50.4	23.6	26.4	26.4	26.6	50.7
llama-3.2-3B	TEAL	—	3.71x	32.2	23.1	49.4	24.1	26.4	26.6	24.6	51.5

Table 3: Model Performance Comparison (acc_norm)

A.2 CWIC Ablations Performance

Model	Technique	Step	Multiplier	all	mmlu	wg	arc:c	HS	arc:e	obqa	piqa
llama-3.2-1B	No Stripes	3000	2.05x	48.0	27.3	54.8	30.7	56.9	57.1	35.8	73.3
llama-3.2-1B	No Stripes	6000	3.05x	45.8	26.2	54.1	30.9	51.4	56.3	32.6	69.1
llama-3.2-1B	No Stripes	9000	4.05x	45.2	26.5	53.1	30.2	49.5	55.7	33.2	68.3
llama-3.2-1B	No Stripes	12000	5.05x	43.9	26.9	51.9	30.7	46.6	52.0	31.6	67.7
llama-3.2-1B	No Stripes	15000	6.05x	42.4	23.6	51.4	28.7	44.6	50.0	31.6	66.7
llama-3.2-1B	No MSE	3000	2.05x	48.3	26.8	54.4	34.0	56.3	59.8	35.2	71.8
llama-3.2-1B	No MSE	6000	3.05x	45.9	24.7	52.2	30.7	51.4	57.4	34.4	70.5
llama-3.2-1B	No MSE	9000	4.05x	45.6	25.8	53.5	29.8	48.9	57.2	35.8	68.1
llama-3.2-1B	No MSE	12000	5.05x	45.0	26.3	56.3	29.2	46.6	53.8	34.0	68.8
llama-3.2-1B	No MSE	15000	6.05x	44.4	24.9	53.2	29.7	46.5	55.1	33.4	68.2

Table 4: Model performance under CWIC ablations

B Hyperparameters

B.1 Settings

Setting	1B	1B noStripes	1B noSTE	3B
Base model	Llama-3.2-1B	Llama-3.2-1B	Llama-3.2-1B	Llama-3.2-3B
Max sequence length	1024	1024	1024	1024
Sequences per step	64	64	64	64
Reduction final	6.05	6.05	6.05	6.05
Reduction warmup length	15,000 steps	15,000 steps	15,000 steps	15,000 steps
Learning rate schedule	1,000,000 steps	1,000,000 steps	1,000,000 steps	1,000,000 steps
Learning rate warmup	500 steps	500 steps	500 steps	500 steps
Learning rate max	0.00005	0.00005	0.00005	0.00003
Optimizer	AdamW	AdamW	AdamW	AdamW
Beta1	0.9	0.9	0.9	0.9
Beta2	0.95	0.95	0.95	0.95
Weight Decay	0.01	0.01	0.01	0.01
β_{dist}	0.99			
α_ϵ	0.1	0.1	0.1	0.1
η_θ	1.0	1.0	1.0	1.0
λ_{APs}	10.0	10.0	10.0	10.0
Stripe Size	512	FULL	512	512

Table 5: Hyperparameter settings for the default model

The hyperparameters for our training run are presented in Table 4. Note that the default training mode had a APR target warmup, normalization, stripe size of 512, and x ste enabled.

B.2 Effects of Hyperparameters

With respect to the bandwidth scale α_ϵ, performance was stable within a range of about 0.05 to 0.25. Values outside of this range caused significant training instability. The choice momentum parameter of running batch statistics β_{dist} (range of 0.9 to 0.99) had very little impact on performance. When calculating pseudo-derivatives, we found that other kernel functions besides the rectangle kernel described in subsection 3.3 gave similar performance.

C Training Data

The training data used in our distillation process is listed below:

- OpenHermes-2.5 Training Data[4] (Teknium, 2023)

[4] teknium/OpenHermes-2.5

- NaturalReasoning Dataset[5] (Yuan et al., 2025)
- SmolTalk Dataset[6] (Allal et al., 2025)
- Orca AgentInstruct-1M-v1[7] (Arindam Mitra, 2024)
- LMSYS-Chat-1M Dataset[8] (Zheng et al., 2023)
- MMLU training split (repeated 5×) (Hendrycks et al., 2021)
- ARC training split (repeated 5×) (Clark et al., 2018)
- WinoGrande training split (repeated 5×) (Sakaguchi et al., 2019)

All data sequences were converted to the standard chat format used by Llama-3.2-1B-Instruct, then filtered for a maximum total sequence length of 1024. We also packed shorter sequences together to increase training efficiency, and used attention masking to prevent interactions between packed sequences.

D Observed Sparsity Patterns

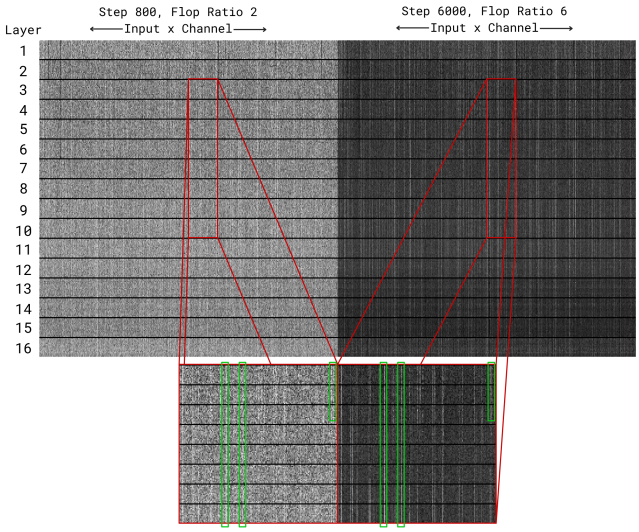

Figure 7: Activation frequencies of W_{gate} at 2x and 6x APR. Rows correspond to weight stripes. Column intensity represents the frequency at which an input position passes the column threshold (darker column indicates lower frequency). The green boxes highlight how important features (brighter columns) emerge early and are magnified in relative importance over training.

[5] facebook/natural_reasoning
[6] HuggingFaceTB/smoltalk
[7] microsoft/orca-agentinstruct-1M-v1
[8] lmsys/lmsys-chat-1m

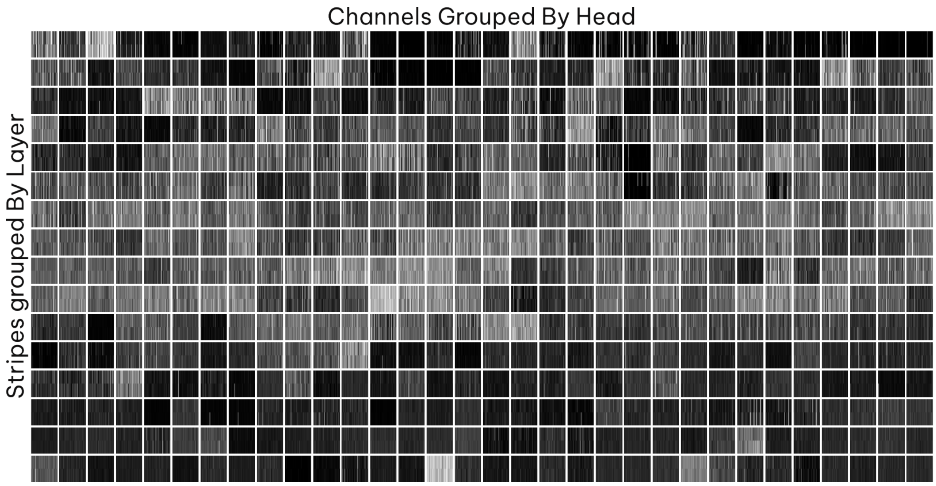

Figure 8: The sparsity levels *within* O heads (each white-outlined cell is a head) tend to be similar across channels. Sometimes entire heads will be always on or always off. Unlike the QKV, UP, GATE, there are no patterns across layers as the input to O is not the residual stream.

E Observed Compute Allocation

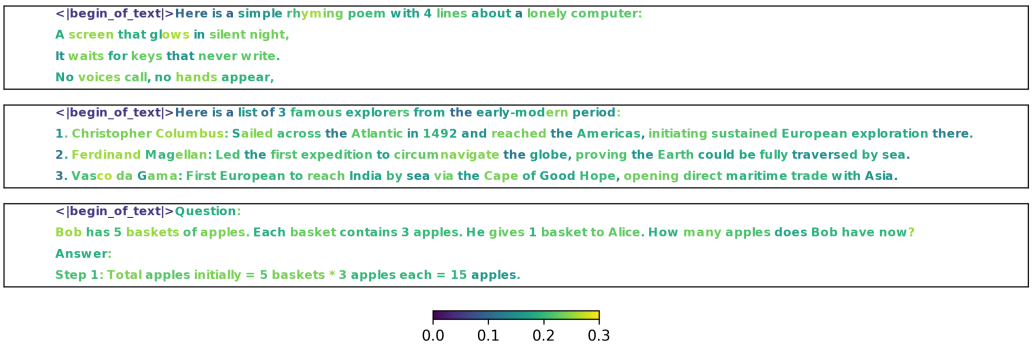

Figure 9: Visualizationx of FLOPs allocated to different tokens during inference for three outut sequences.