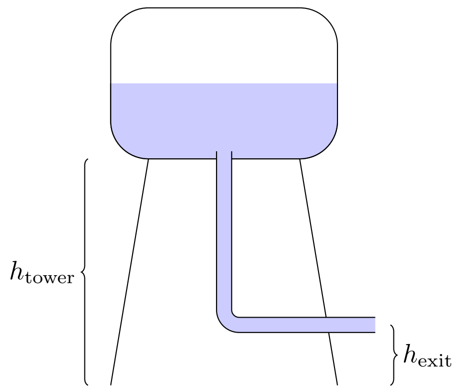
Model water tower

Jackson constructs a model of a water tower, connecting the bottom of the reservoir to a pipe that expels the water elsewhere. The water exits at a height (h_{exit}) that is smaller than the height of the tower (h_{tower}). The pressure at the bottom of the reservoir is fixed ($P_{\text{reservoir}}$), and is at least as large as atmospheric pressure. The pressure at the exit is atmospheric. The pipe has a constant area (A), and some resistance (R). Jackson can modify the model by changing the heights, the reservoir pressure, the area, and the resistance.



(a) Which of the following modifications would double the flow rate within the pipe?

○ doubling h_{tower} ○ halving h_{exit} ○ doubling $P_{\text{reservoir}}$ ○ halving A ⊗ halving R

Using the Energy Density Equation to compare the point at the bottom of the reservoir to the point where the water exits the pipe, one finds

$$(P_{\text{atm}} - P_{\text{reservoir}}) + \rho g(h_{\text{exit}} - h_{\text{tower}}) = -IR.$$

Solving for I, one finds

$$I = \frac{(P_{\text{reservoir}} - P_{\text{atm}}) + \rho g(h_{\text{tower}} - h_{\text{exit}})}{R}.$$

Though all options listed would increase I, only halving R would exactly double it.

(b) Which of the following modifications would increase the speed at which water exits the pipe?

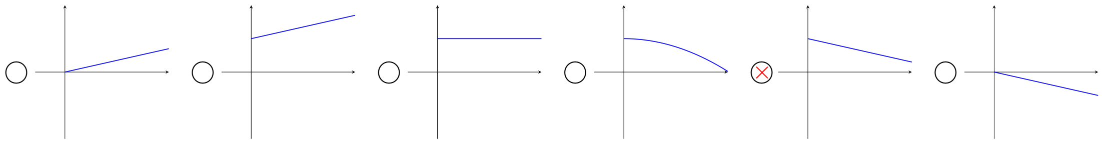
☒ increasing h_{tower} ☐ increasing h_{exit} ☒ increasing $P_{\text{reservoir}}$ ☐ increasing A ☐ increasing R

Increasing the height of the tower or the pressure of the reservoir increases the flow rate by increasing the amount of energy that ΔE_{th} must account for; with the increased flow rate, the speed must be higher as well.

Increasing the area, while keeping the flow rate constant, would decrease the speed. The remaining options decrease the flow rate, and so would decrease the speed as well.

Energy Density Equation	**Continuity Equation**	**Constants**
$\Delta P + \frac{\Delta PE}{\text{vol.}} + \frac{\Delta KE}{\text{vol.}} = \frac{E_{\text{pump}}}{\text{vol.}} - \frac{\Delta E_{\text{th}}}{\text{vol.}}$ $\Delta P + \rho g \Delta h + \frac{1}{2}\rho\Delta(v^2) = E_{\text{pump}}/\text{vol.} - IR$	$I = Av$ $I_{\text{in}} = I_{\text{out}}$	$\rho_{\text{water}} = 1000 \text{ kg/m}^3$ $g = 10 \text{ m/s}^2$ $P_{\text{atm}} = 1 \text{ atm} = 10^5 \text{ Pa OR J/m}^3 \text{ OR kg/(m} \cdot \text{s}^2)$

(c) Suppose $P_{\text{reservoir}} = P_{\text{atm}}$. Which of the following is plots could depict the flow rate on the vertical axis plotted against the exit height on the horizontal axis, with all other parameters held fixed?



Similar to the previous parts, one may recognize that increasing the exit height should decrease the flow rate: with the fixed resistance, decreasing the change in energy density implies that the flow rate must go down so the thermal energy change matches. When the exit height is zero, there is a pressure difference and so there must be some non-zero flow rate. This leaves only the fourth and fifth options, of which the fifth is correct as the formulas for energy density are all linear in height.

One can also identify these properties directly from solving for the flow rate as done above,

$$I = \frac{(P_{\text{reservoir}} - P_{\text{atm}}) + \rho g(h_{\text{tower}} - h_{\text{exit}})}{R},$$

which is a linear function of h_{exit} with negative slope and non-zero y-intercept.

(d) With $P_{\text{reservoir}} = 2P_{\text{atm}}$, $h_{\text{tower}} = 2h_{\text{exit}} = 20$ m, $A = 1$ m^2, and $R = 10^6$ J $\cdot$ s/m^6, determine the speed of the water.

$v =$ [0.2] m/s

Using Bernoulli's equation, $(P_{\text{atm}} - P_{\text{reservoir}}) + \rho g(h_{\text{exit}} - h_{\text{tower}}) = -IR$ one finds

$$I = (-10^5 + 1000 \cdot 10 \cdot (-10))/(-10^6) = 0.2 \text{ m}^3/\text{s}.$$

Then, $v = I/A = 0.2$ m/s.

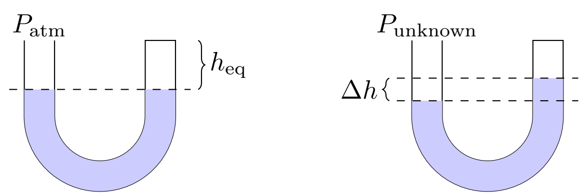
Energy Density Equation	Continuity Equation	Constants
$\Delta P + \frac{\Delta PE}{\text{vol.}} + \frac{\Delta KE}{\text{vol.}} = \frac{E_{\text{pump}}}{\text{vol.}} - \frac{\Delta E_{\text{th}}}{\text{vol.}}$	$I = Av$	$\rho_{\text{water}} = 1000$ kg/m^3
$\Delta P + \rho g \Delta h + \frac{1}{2}\rho\Delta(v^2) = E_{\text{pump}}/\text{vol.} - IR$	$I_{\text{in}} = I_{\text{out}}$	$g = 10$ m/s^2
		$P_{\text{atm}} = 1$ atm $= 10^5$ Pa OR J/m^3 OR kg/(m $\cdot$ s^2)

Unusual Barometer

An unusual barometer (i.e. device for measuring pressure) is constructed by having a U-shaped pipe that is open on the left end, and closed on the right end. On the open end, water is exposed to the pressure of the room it is in. On the closed end, some fixed amount of an ideal gas is stored at a constant temperature. Recall that the volume of an ideal gas is inversely proportional to pressure, e.g. doubling the pressure requires the volume to be halved.

The pipe has a constant area. The amount of water, and the amount of air in the closed side, is the same in the following scenarios:

- Scenario A: When the room is at atmospheric pressure, the water levels are equal, with h_{eq} being the height of the air column on the closed side.
- Scenario B: When the room is at some unknown pressure (P_{unknown}), a difference in the height of the water levels (Δh) is observed.



(a) What relationship does P_{unknown} have with P_{atm}?

○ $P_{\text{unknown}} < P_{\text{atm}}$ ○ $P_{\text{unknown}} = P_{\text{atm}}$ ⊗ $P_{\text{unknown}} > P_{\text{atm}}$ ○ cannot be determined

In Scenario A, using the energy density equation to compare the pressures at the surface of the water levels between the left and right sides, one determines that $P_{\text{atm}} = P_{A,\text{right}}$. In Scenario B, the same comparison determines that $P_{\text{unknown}} = P_{B,\text{right}} + \rho g \Delta h$. Since the volume of the air on the right side is smaller in Scenario B, one knows that $P_{B,\text{right}} > P_{A,\text{right}}$. Consequently,

$$P_{\text{unknown}} = P_{B,\text{right}} + \rho g \Delta h > P_{B,\text{right}} > P_{A,\text{right}} = P_{\text{atm}},$$

so it is definitely larger than atmospheric.

In this case, one's intuition would also serve them well, the pressure on the open side needs to be higher to 'push the water down'.

(b) Suppose, starting with Scenario A, extra water is poured into the left side. Which of the following would be true when equilibrium is reached:

☒ The left water level is higher than before.

☒ The right water level is higher than before.

☐ The water level on the left still matches the water level on the right.

☐ The pressure of the air in the closed side is still atmospheric.

Energy Density Equation	**Continuity Equation**	**Constants**
$\Delta P + \frac{\Delta PE}{\text{vol.}} + \frac{\Delta KE}{\text{vol.}} = \frac{E_{\text{pump}}}{\text{vol.}} - \frac{\Delta E_{\text{th}}}{\text{vol.}}$ $\Delta P + \rho g \Delta h + \frac{1}{2}\rho\Delta(v^2) = E_{\text{pump}}/\text{vol.} - IR$	$I = Av$ $I_{\text{in}} = I_{\text{out}}$	$\rho_{\text{water}} = 1000\ \text{kg/m}^3$ $g = 10\ \text{m/s}^2$ $P_{\text{atm}} = 1\ \text{atm} = 10^5$ Pa OR J/m^3 OR $\text{kg/(m}\cdot\text{s}^2)$

Since more water is added, at least one of the water levels must go up. If only the left water level went up, then the energy density at the surface of the left water level would increase without the same increase happening to the right side. If only the right water level went up, then the energy density of the right water level would increase without the same increase happening to the left side. Since the energy densities must remain equal, that means that both water levels must go up.

Since the right water level went up, the pressure on the right side is now higher than atmospheric. In order for the left side to have an equal increase in energy density despity the pressure not rising, its water level must have risen further. Consequently, the water level on the left will no longer match the water level on the right.

(c) With $h_{\text{eq}} = \Delta h = 1$ m, one may deduce that the volume of the air in the closed side of the pipe is twice as large in Scenario A as it is in Scenario B.
Use this information to determine P_{unknown}.

$P_{\text{unknown}} =$ [2.1] $\times 10^5$ Pa

Using the Energy Density Equation, one may determine that

$$P_{\text{unknown}} = P_{B,\text{right}} + \rho g \Delta h.$$

Recalling as in part (a) that $P_{A,\text{right}} = P_{\text{atm}}$, with the volume of air halved in Scenario B, the pressure must be twice as high: $P_{B,\text{right}} = 2P_{\text{atm}}$. So, plugging in the values

$$P_{\text{unknown}} = 2P_{\text{atm}} + 1000 \cdot 10 \cdot 1 \text{ Pa} = 2 \times 10^5 + 10^4 \text{ Pa} = 2.1 \times 10^5 \text{ Pa}.$$

Energy Density Equation	**Continuity Equation**	**Constants**
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