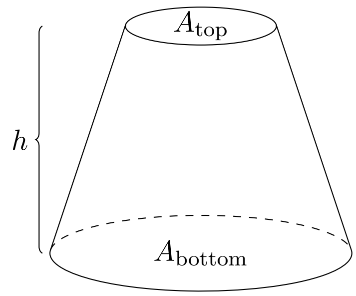
Quiz 1 (compiled : 2026/04/16 at 01:56; version 1)

Name: ____________________ **Student ID:** ____________________

Problem 1: A section of a vertical pipe of height h with resistance R has a cross-sectional area that changes. The top area (A_{top}) is smaller than the bottom area (A_{bottom}). For a particular flow direction and flow rate I_0, it is observed that the pressure at the top (P_{top}) is equal to the pressure at the bottom (P_{bottom}).



(a) For this part, suppose the top and bottom areas are actually equal. With the pipe's flow set up to have equal pressures at the top and bottom, and given $h = 2$ m, $R = 5 \times 10^4$ Js/m^6, and $\rho = 1000$ kg/m^3, determine the magnitude and direction of I_0. (Round to two decimal places.)

$I_0 =$ 0.40 m^3/s

⊗ flowing down ○ flowing up ○ cannot be determined

Seeing the problem start, a question that one may immediately ask themselves is 'What is this particular flow direction and flow rate that is mentioned?'. After all, one would usually be surprised to have the pressures equal after so many other things change.

The flow direction affects only the thermal energy change, so we can figure that out first. The potential and kinetic energies are lower at the bottom; therefore, the thermal energy must be higher there. (This is exactly what you did in DL Activity 5.3(B), except pressure is replaced by thermal energy.) In order for the thermal energy to be higher at the bottom, that is where the water must flow.

To compute the flow rate, one must set up the equation, which I chose to make simpler by removing the kinetic term by specifying that the area stays constant for this part. Then, the equation is simply

$$\rho g(h_{\text{bottom}} - h_{\text{top}}) = -IR \implies I = \frac{\rho g h}{R}.$$

(b) Suppose the pipe is set up with the flow direction and flow rate that made the top and bottom pressures equal. Then, one of the following changes are made. For which changes does P_{bottom} end up larger than P_{top}?

(Changing the height h does not change the resistance R.)

☒ increasing A_{bottom} ☒ increasing h ☐ increasing I_0 ☐ increasing R ☒ reversing the direction of I_0

As I pointed out in lecture (see 'Lecture 2 - Lecturing' slide 6 and 8 for my 'suggested questions'), a very important part of understanding a physical system is to identify how the system would respond to changes.

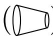
Seeking to make the pressure at the bottom bigger than the pressure at the top one needs to either: decrease the bottom's potential energy, decrease the bottom's kinetic energy, or decrease the bottom's thermal energy. This means that one needs to either increase the height difference, increase the bottom area, or decrease the flow rate or resistance. In addition the thermal energy is affected by the direction of the flow rate; by reversing it, the top is the one which will have a higher thermal energy, so the bottom is the one with the higher pressure.

If one is more mathematically inclined, one can of course solve for $(P_{\text{bottom}} - P_{\text{top}})$,

$$(P_{\text{bottom}} - P_{\text{top}}) = \rho g h - IR + \frac{1}{2}\rho I^2 \left(\frac{1}{A_{\text{bottom}}^2} - \frac{1}{A_{\text{top}}^2} \right),$$

from which one can track what makes the LHS become more positive.

(c) In each of the following cases, the pipe has been rotated in some way. If possible, determine the direction that the fluid must flow to keep the pressures on the two ends equal:

- Rotated 90 degrees clockwise ():
  ⊗ flowing left ○ flowing right ○ flowing down ○ flowing up ○ cannot be determined
- Flipped over ():
  ○ flowing left ○ flowing right ○ flowing down ○ flowing up ⊗ cannot be determined
- Rotated 45 degrees counter-clockwise ():
  ○ flowing down-left ⊗ flowing down-right ○ flowing up-left ○ flowing up-right ○ cannot be determined
- Rotated 135 degrees clockwise ():
  ○ flowing down-left ○ flowing down-right ○ flowing up-left ○ flowing up-right ⊗ cannot be determined

Unlike the previous parts, I chose this question as it is something one might not predict from the problem start. After all, part of your physics knowledge is the ability to 'understand novel scenarios' ('Lecture 1' slide 3).

However, if you were able to deduce the flow direction for the unrotated pipe, you can reuse the logic it for any pipe with changing heights/areas.

Since the pressure is held fixed, the thermal energy should be higher where the other energies are lower. This means that the water should flow towards the larger area / lower height.

This is sufficient for two of the pipes: the one which is horizontal (so it flows to the left towards the larger area), and the one that was only rotated 45 degrees (where it is flowing down and to the right towards the larger opening).

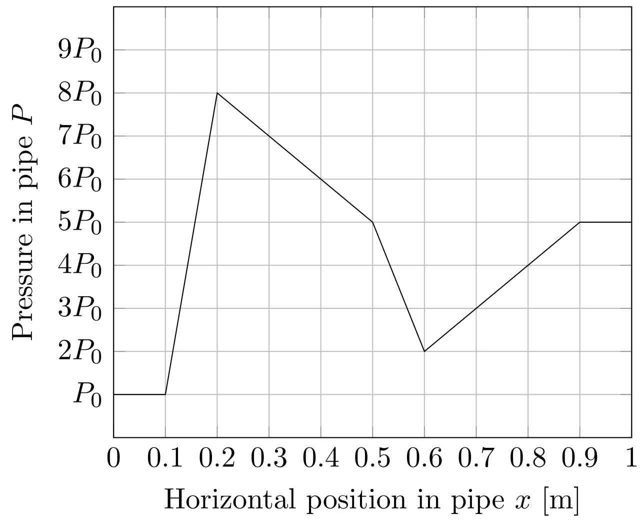
But what about the pipes where this information disagrees? For the pipe that is flipped over and the pipe that was rotated 135 degrees clockwise, the larger area is at the larger height. In this case, the comparison cannot be determined. Much like in part (a), you should recognize this from what you have done before: DL1 Activity 5.1(B) was precisely such a pipe, and my example in lecture ('Lecture 2 - Lecturing' slide 13) demonstrated a scenario with a similar uncertainty.

If one is more mathematically inclined, one could identify that the direction of the current is related to thermal energy change, and solve for it as

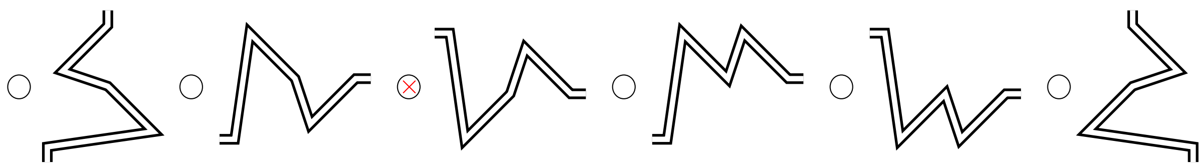
$$E_{\text{thermal},1}/\text{vol.} - E_{\text{thermal},2}/\text{vol.} = (PE_1/\text{vol.} - PE_2/\text{vol.}) + (KE_1/\text{vol.} - KE_2/\text{vol.})$$

Problem 2: A section of pipe traverses 1 meter horizontally, with water flowing from left to right inside of it. The pipe can change heights and/or areas, but has no pumps or resistance. The left end of the pipe (at $x = 0$ m) has a fixed pressure P_0. The pressures elsewhere in the pipe for a flow rate of 1 m^3/s are plotted below.

Pressure measurements for $I = 1$ m^3/s



(a) Suppose the area of the pipe is constant. Which of the following could be the shape of the pipe:



This problem start is unusual compared to most DL material, as we usually use a pipe's setup to predict pressures, rather than the other way around. However, with the problem start titled 'lost blueprints', it is only natural to ask what the blueprint for such a pipe could be.

To avoid mixing heights and areas, I focused on just one: larger pressures should have lower potential energies and so lower heights. Consequently, the side-view of the pipe should just be the pressure graph 'flipped'.

If one is more mathematically inclined, then you could solve for the height as

$$h(x) = -\frac{P(x)}{\rho g} + \left(h_0 + \frac{P_0}{\rho g}\right),$$

and see that the function needs to be flipped.

(b) Suppose the height of the pipe is constant, the initial area at $x = 0$ is $A_0 = 0.15$ m^2, and the initial pressure is $P_0 = 1.2$ kPa. Determine the location of the maximum area of the pipe. (Round to two decimal places.)

$x_{\text{where } A_{\max}} =$ 0.2 m

Determine the maximum area. (Round to three decimal places.)
(You may find $\Delta(v^2) = v_f^2 - v_i^2 = I^2/A_f^2 - I^2/A_i^2 = I^2\Delta(1/A^2)$ helpful.)

$A_{\max} =$ 0.190 m^2

When I originally wrote it, this question included only the second half of computing the maximum area. However, I thought this may be too challenging, as it requires two steps: to first identify where the maximum area even occurs, and then to compute it. So, I split it into two halves to help guide you towards the approach.

A larger area corresponds to a smaller kinetic energy, and so to a larger pressure since the height is fixed. Therefore, the *largest* area should be at the *largest* pressure. Thus, $x = 0.2$ m. (I apologize to those who were thrown off by being asked to round to two decimal places, but I did not want to give away immediately that it was on a grid-line by saying 'round to 1 decimal place' or leave it ambiguous by writing nothing.)

The second half, even with the guidance to find the location, was too challenging. Setting up the equation to solve

$$(P_{\max} - P_0) + \frac{1}{2}\rho I^2\left(\frac{1}{A_{\max}^2} - \frac{1}{A_0^2}\right) = 0,$$

one either plugs the known values in directly, or rearranges first to find

$$A_{\max} = \sqrt{\frac{1}{\frac{1}{A_{\max}^2} - \frac{2}{\rho}(P_0 - P_{\max})}}.$$

This is an admittedly large expression to throw into a calculator, and very few students made it to the finish line. Those students are rewarded by some extra points, and the rest should not worry as it means the curve is that much more generous.

(c) Determine whether the following are true, false, or not possible to determine with the given information:

- If the height of the pipe is constant, the areas at $x = 0.5$ m and $x = 0.9$ m are the same.
  ⊗ True ◯ False ◯ Not possible to determine
- If the height of the pipe is constant, the areas at $x = 0$ m and $x = 1$ m are the same.
  ◯ True ⊗ False ◯ Not possible to determine
- If the height of the pipe is constant, then increasing the flow rate would increase the pressure differences.
  ⊗ True ◯ False ◯ Not possible to determine
- If the area of the pipe is constant, then increasing the flow rate would increase the pressure differences.
  ◯ True ⊗ False ◯ Not possible to determine

For the questions asking to compare areas, it is the same idea as before: larger pressures require larger areas when at the same height. If the pressures are equal (like at $x = 0.5$ m and $x = 0.9$ m), then the areas are equal, and if they are different (like at $x = 0$ m and $x = 1$ m), then they are the same.

For the other two parts, they are to test a distinction between potential and kinetic energies that we have talked about ever since the first DL (Activity 5.1(B)(5)): potential energies do not depend on flow rate, but kinetic energies do.

If the height is fixed, then kinetic energy change is responsible for pressure change, so it is affected by the flow rate. If the area is fixed, then potential energy change is responsible for pressure change, so it is not affected by the flow rate.

If you are more mathematically inclined, then you could write one of

$$(P_2 - P_1) + \rho g(h_2 - h_1) = 0$$

$$(P_2 - P_1) + \frac{1}{2}\rho I^2\left(\frac{1}{A_2^2} - \frac{1}{A_1^2}\right) = 0$$

depending on what was held constant, and solve from there.