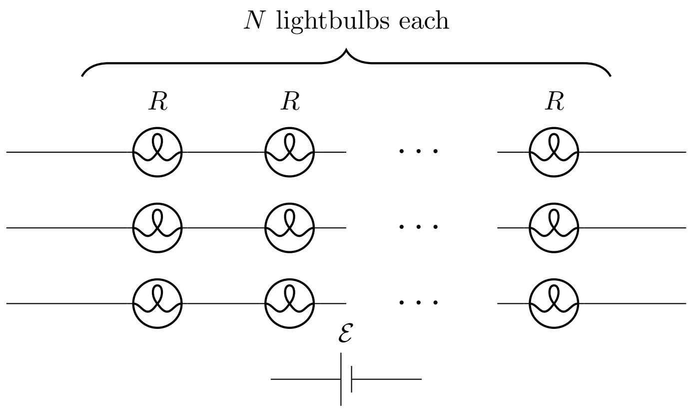
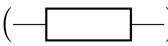
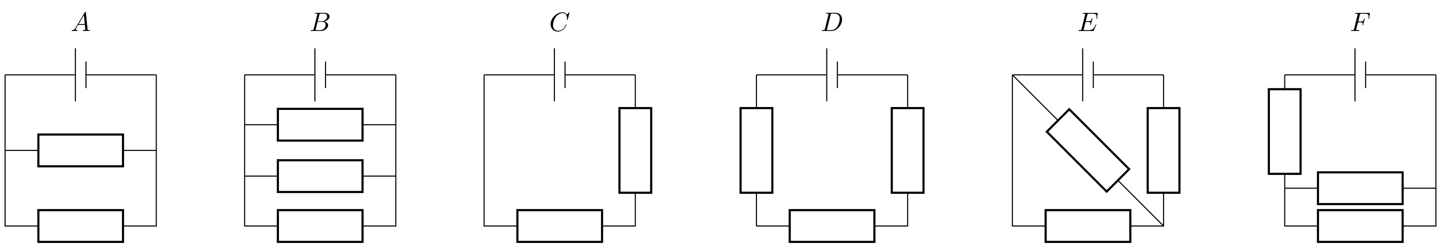
Quiz 2 (compiled : 2026/04/29 at 21:38; version 1)

Name: 　　　　　　　　　　　　 **Student ID:**

Problem 1: A set of light strips is sold with three strips and one battery to power them. Each light strip consists of N identical light bulbs, each lightbulb having resistance R, attached one after another on a wire as shown in the figure below. The battery supplies a voltage $\mathcal{E}$. The ends of the light strips can be connected to each other, or to either end of the battery. A designer is thinking of the different ways the strips can be wired together.



Using () to represent each of the strips, the designer considers the following designs, some of which don't use all three strips:



Before getting into the specific questions, one should consider what could have been garnered just from the problem start. We are given 4 components: three strips each of resistance NR, and one battery with voltage $\mathcal{E}$. We are told that they can be, and indeed will be, wired together. It is only natural to consider the designs, most of which are listed above. Once one has a single battery and a group of resistors, there is only one thing to do: compute the equivalent resistance and use that for whatever one might want to figure out.

In circuit A, the two strips are in parallel, so the equivalent resistance is

$$R_{\text{eq},A} = \frac{1}{\frac{1}{NR} + \frac{1}{NR}} = \frac{1}{2}NR.$$

In circuit B, the three strips are in parallel, so the equivalent resistance is

$$R_{\text{eq},B} = \frac{1}{\frac{1}{NR} + \frac{1}{NR} + \frac{1}{NR}} = \frac{1}{3}NR.$$

In circuit C, the two strips are in series, so the equivalent resistance is

$$R_{\text{eq},C} = NR + NR = 2NR.$$

General Circuits	**Equivalent Resistance**	**Exponential Decay**
$\Delta V_{\text{loop}} = 0,\ I_{\text{in}} = I_{\text{out}},\ \Delta V_{\text{battery}} = \mathcal{E}$ $\Delta V_{\text{resistor}} = -IR,\ \Delta V_{\text{capacitor}} = Q/C$ $P = \lvert I\Delta V\rvert = (\Delta V)^2/R = I^2R$	$R_{\text{series}} = R_1 + R_2 + \cdots$ $R_{\text{parallel}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$	$\frac{dy}{dt} = \lambda(y_{\text{eq}} - y)$ $\implies y = (y_0 - y_{\text{eq}})e^{-\lambda t} + y_{\text{eq}}$ $\tau = t_{1/e} = 1/\lambda = RC,\ t_{1/2} = \ln(2)t_{1/e}$

In circuit D, the three strips are in series, so the equivalent resistance is

$$R_{\text{eq},D} = NR + NR + NR = 3NR.$$

In circuit E, there are two resistors in parallel, followed by one in series, so the equivalent resistance is

$$R_{\text{eq},E} = \frac{1}{\frac{1}{NR} + \frac{1}{NR}} + NR = \frac{3}{2}NR.$$

In circuit F, there are two resistors in parallel, preceded by one in series, so the equivalent resistance is

$$R_{\text{eq},F} = NR + \frac{1}{\frac{1}{NR} + \frac{1}{NR}} = \frac{3}{2}NR.$$

(Places where similar computations came up in DLs: FNT 1 at the end of DL3, Activity 5.12 in DL4, and the Mock Quiz from DL5.)

If you did not come to the quiz having worked this out, or at least thought about how you would work it out, I would unfortunately say you did not sufficiently analyze the problem start.

(a) For each of the following pairs of designs, compare the total power of the lightbulbs involved. *(Hint: The power for a group of components can be computed using equivalent resistance,* $P = (\Delta V)^2/R_{eq} = I^2 R_{eq}$*)*

- A and B : ○ $P_A > P_B$ ⊗ $P_A < P_B$ ○ $P_A = P_B$ ○ Cannot be determined
- C and D : ⊗ $P_C > P_D$ ○ $P_C < P_D$ ○ $P_C = P_D$ ○ Cannot be determined
- B and E : ⊗ $P_B > P_E$ ○ $P_B < P_E$ ○ $P_B = P_E$ ○ Cannot be determined
- E and F : ○ $P_E > P_F$ ○ $P_E < P_F$ ⊗ $P_E = P_F$ ○ Cannot be determined

It occured to me we did not explicitly cover how computing power interacts with computing equivalent resistance, hence I gave a hint to make sure that was something you're aware works as one might naively expect.

In this case, the voltage across the equivalent resistor is always $\mathcal{E}$, so the only thing changing is equivalent resistance. Using the formula for power in terms of voltage and resistance, one finds that a higher equivalent resistance leads to a lower power.

From this, one directly deduces

$$\begin{aligned} R_{\text{eq},A} > R_{\text{eq},B} &\implies P_A < P_B \\ R_{\text{eq},C} < R_{\text{eq},D} &\implies P_C > P_D \\ R_{\text{eq},B} < R_{\text{eq},E} &\implies P_B > P_E \\ R_{\text{eq},E} = R_{\text{eq},F} &\implies P_E = P_F \end{aligned}$$

(b) Suppose $\mathcal{E} = 120$ V, $R = 10\ \Omega$, and $N = 10$, determine the magnitude of the current through the battery in design E. (Round to 2 decimal places)

$|I_{E,\text{bat}}| =$ 0.80 A

This comes from a direct computation, e.g. starting with a loop rule

$$\mathcal{E} - IR_{\text{eq},E} = 0 \implies I = \frac{\mathcal{E}}{\frac{3}{2}NR} = \frac{2\mathcal{E}}{3NR} = \frac{2 \cdot 120}{3 \cdot 10 \cdot 10}\ \text{A} = 0.80\ \text{A}.$$

General Circuits	Equivalent Resistance	Exponential Decay
$\Delta V_{\text{loop}} = 0,\ I_{\text{in}} = I_{\text{out}},\ \Delta V_{\text{battery}} = \mathcal{E}$ $\Delta V_{\text{resistor}} = -IR,\ \Delta V_{\text{capacitor}} = Q/C$ $P = \lvert I\Delta V\rvert = (\Delta V)^2/R = I^2R$	$R_{\text{series}} = R_1 + R_2 + \cdots$ $R_{\text{parallel}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$	$\frac{dy}{dt} = \lambda(y_{\text{eq}} - y)$ $\implies y = (y_0 - y_{\text{eq}})e^{-\lambda t} + y_{\text{eq}}$ $\tau = t_{1/e} = 1/\lambda = RC,\ t_{1/2} = \ln(2)t_{1/e}$

(c) Select all expressions that could be a magnitude of the voltage difference across an individual lightbulb in one of the designs above:

☐ $\frac{2\mathcal{E}}{N}$ ☒ $\frac{\mathcal{E}}{N}$ ☒ $\frac{\mathcal{E}}{2N}$ ☐ $\mathcal{E}$ ☐ $\frac{\mathcal{E}}{2}$ ☐ $\frac{\mathcal{E}}{3}$ ☒ $\frac{\mathcal{E}}{3N}$ ☒ $\frac{2\mathcal{E}}{3N}$ ☐ $\frac{\mathcal{E}}{4N}$ ☐ $\frac{\mathcal{E}}{4}$

This is the only part that does not have an immediate solution from finding equivalent resistances, and is the only part I'd say shouldn't have been 'expected' given a problem start about lightbulbs.

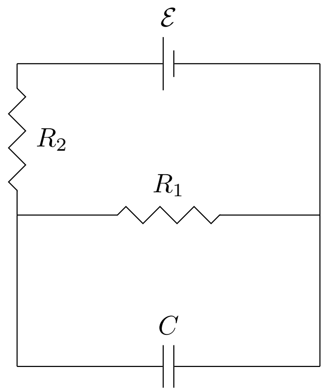
First, one finds the possible voltage differences across a whole strip. In designs A and B, the voltage differences across each strip are $\mathcal{E}$. In designs C and D, the voltage difference is split between the resistors, having $\mathcal{E}/2$ and $\mathcal{E}/3$ per strip.

Designs E and F are more subtle. The current splits in half for the pair of resistors in parallel, so the voltage drop across the parallel part is half of the voltage drop across the resistor in series with the battery. Since the total voltage drop is $\mathcal{E}$, it must be split into $\mathcal{E}/3$ for the parallel resistors and $2\mathcal{E}/3$ for the resistor in series with the battery.

But you were asked about individual lightbulbs, rather than whole strips. With N lightbulbs in series on each strip, the voltage difference is split between them, so one divides the found values by N. If one made only this observation and eliminated the answers that did not depend on N, that would already be enough to get 60% of the credit for the question by marking 4 correct and 2 incorrect answers that involve N.

General Circuits	**Equivalent Resistance**	**Exponential Decay**
$\Delta V_{\text{loop}} = 0,\ I_{\text{in}} = I_{\text{out}},\ \Delta V_{\text{battery}} = \mathcal{E}$	$R_{\text{series}} = R_1 + R_2 + \cdots$	$\frac{dy}{dt} = \lambda(y_{\text{eq}} - y)$
$\Delta V_{\text{resistor}} = -IR,\ \Delta V_{\text{capacitor}} = Q/C$	$R_{\text{parallel}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$	$\implies y = (y_0 - y_{\text{eq}})e^{-\lambda t} + y_{\text{eq}}$
$P = \lvert I\Delta V\rvert = (\Delta V)^2/R = I^2R$		$\tau = t_{1/e} = 1/\lambda = RC,\ t_{1/2} = \ln(2)t_{1/e}$

Problem 2: A capacitor with capacitance C is wired 'in parallel' with a resistor R_1. If the capacitor starts discharged, then the battery with voltage difference $\mathcal{E}$ would charge it.



Seeing a circuit with multiple loops and a capacitor, two thoughts should pass through ones mind: I should write down loop/junction rules, and I should figure out the initial and equilibrium values.

With the junction rule $I_{\text{battery}} = I_{R_2} = I_{R_1} + I_C$, one finds the loop rules for the top and outer loop

$$\mathcal{E} - I_{R_1}R_1 - (I_{R_1} + I_C)R_2 = 0,$$
$$\mathcal{E} - Q/C - (I_{R_1} + I_C)R_2 = 0.$$

Initially, the capacitor is discharged, so $Q = 0$. This leaves two equations with two unknowns, which can be solved to find

$$I_{R_1} = 0,\ I_C = I_{R_2} = I_{\text{battery}} = \mathcal{E}/R_2,$$

which one should expect as the branch with the capacitor has no resistance, so all the current goes around through that branch and through resistor R_2. Resistor R_1 is not yet involved.

At equilibrium, the capacitor is charged, so $I_C = 0$. This makes the first equation have only one unknown, which can be solved for

$$I_{R_1} = I_{R_2} = I_{\text{battery}} = \frac{\mathcal{E}}{R_1 + R_2}.$$

One could further use these solutions to make deductions about voltage differences, or make plots, but I will comment on that once we get to it below.

If you did not come to the quiz having worked out the initial/equilibrium conditions, or at least thought about how you would work it out, I would unfortunately say you did not sufficiently analyze the problem start. In fact, every part (except perhaps the time constant, which I will comment on below) is something that one could have deduced before coming to the quiz.

(a) At the moment when the capacitor is still discharged, what is the current through each of the components? *(Hint: Capacitor discharged means $Q = 0$.)*

- Battery : ○ 0 ○ $\mathcal{E}\frac{R_1R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1+R_2}{R_1R_2}$ ○ $\frac{\mathcal{E}}{R_1}$ ⊗ $\frac{\mathcal{E}}{R_2}$ ○ $\frac{\mathcal{E}}{R_1+R_2}$
- Resistor 1 : ⊗ 0 ○ $\mathcal{E}\frac{R_1R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1+R_2}{R_1R_2}$ ○ $\frac{\mathcal{E}}{R_1}$ ○ $\frac{\mathcal{E}}{R_2}$ ○ $\frac{\mathcal{E}}{R_1+R_2}$
- Resistor 2 : ○ 0 ○ $\mathcal{E}\frac{R_1R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1+R_2}{R_1R_2}$ ○ $\frac{\mathcal{E}}{R_1}$ ⊗ $\frac{\mathcal{E}}{R_2}$ ○ $\frac{\mathcal{E}}{R_1+R_2}$
- Capacitor : ○ 0 ○ $\mathcal{E}\frac{R_1R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1+R_2}{R_1R_2}$ ○ $\frac{\mathcal{E}}{R_1}$ ⊗ $\frac{\mathcal{E}}{R_2}$ ○ $\frac{\mathcal{E}}{R_1+R_2}$

This directly follows from the solutions for the current given above.

General Circuits	**Equivalent Resistance**	**Exponential Decay**
$\Delta V_{\text{loop}} = 0,\ I_{\text{in}} = I_{\text{out}},\ \Delta V_{\text{battery}} = \mathcal{E}$	$R_{\text{series}} = R_1 + R_2 + \cdots$	$\frac{dy}{dt} = \lambda(y_{\text{eq}} - y)$
$\Delta V_{\text{resistor}} = -IR,\ \Delta V_{\text{capacitor}} = Q/C$	$R_{\text{parallel}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$	$\implies y = (y_0 - y_{\text{eq}})e^{-\lambda t} + y_{\text{eq}}$
$P = \lvert I\Delta V\rvert = (\Delta V)^2/R = I^2R$		$\tau = t_{1/e} = 1/\lambda = RC,\ t_{1/2} = \ln(2)t_{1/e}$

(b) After a long time, when the capacitor is fully charged, what is the magnitude of the voltage difference across each component? *(Hint: Capacitor fully charged means $I_{capacitor} = 0$)*

- Battery : ○ 0 ⊗ $\mathcal{E}$ ○ $\mathcal{E}\frac{R_1}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1}{R_2}$ ○ $\mathcal{E}\frac{R_2}{R_1}$
- Resistor 1 : ○ 0 ○ $\mathcal{E}$ ⊗ $\mathcal{E}\frac{R_1}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1}{R_2}$ ○ $\mathcal{E}\frac{R_2}{R_1}$
- Resistor 2 : ○ 0 ○ $\mathcal{E}$ ○ $\mathcal{E}\frac{R_1}{R_1+R_2}$ ⊗ $\mathcal{E}\frac{R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1}{R_2}$ ○ $\mathcal{E}\frac{R_2}{R_1}$
- Capacitor : ○ 0 ○ $\mathcal{E}$ ⊗ $\mathcal{E}\frac{R_1}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_2}{R_1+R_2}$ ○ $\mathcal{E}\frac{R_1}{R_2}$ ○ $\mathcal{E}\frac{R_2}{R_1}$

The voltage for a battery does not change, so it is still $\mathcal{E}$. I was R to see that only $\tilde{6}0\%$ of the students answered this correctly, as the figure labels the battery with $\mathcal{E}$ and even the formula sheet states $\Delta V_{\text{battery}} = \mathcal{E}$. Please review Activity 5.17 of DL7 for how voltages work in a charging circuit, where you used the constant voltage provided by the battery to deduce the plots for the capacitor's and resistor's voltages.

For the resistors, one can use $|\Delta V| = |IR|$ to use the solutions for the current to find their voltages.

The capacitor is in parallel with resistor 1, as stated in the problem start, and so has the same voltage.

(c) Suppose $\mathcal{E} = 120$ V, $R_1 = 10\ \Omega$, $R_2 = 10\ \Omega$, and $C = 5$ F.
What is the half-life corresponding to the charging process? (Round to two decimal places)

$t_{1/2} =$ [See soln.] s

Though I did not realize this at the time of writing the quiz, this question is a more subtle than I expected. Given the conventional wisdom shared in lecture and in DL, one would expect $\tau = R_2 C$, since the loop through which the capacitor is charged is the one only with R_2. However, this turns out to be wrong, and one must be careful to find that $\tau = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}} C$ instead. (To get the half-life, one simply multiplies by $\ln(2)$ as on the formula sheet.)

Given the question's difficulty departed from that of the lecture and DL material, I have made the following decision. This question is excluded from the grading for all students, and the grades will be curved as though it did not exist. However, to reward those students who approached it the intended way (using R_2) or the correct way (using $\frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$), they will receive +1 point. I hope you agree that this compromise is more fair than dropping the question entirely, or grading it directly. (Of course, the fairest option would have been for the question not to exist in the first place, but it is too late for that. I apologize for the oversight.)

Here are three ways to reach the correct answer in increasing order of efficiency:

1. *Rearranging system of equations to find 'exponential decay' expression*
   The system of equations above can be rearranged as

$$\begin{cases} \mathcal{E} - I_{R_1}R_1 - (I_{R_1} + I_C)R_2 = 0, \\ \mathcal{E} - Q/C - (I_{R_1} + I_C)R_2 = 0. \end{cases} \rightarrow \begin{cases} I_{R_1} = \dfrac{\mathcal{E} - I_C R_2}{R_1 + R_2}, \\ \mathcal{E} - Q/C - (I_{R_1} + I_C)R_2 = 0. \end{cases} \rightarrow \begin{cases} I_{R_1} = \dfrac{\mathcal{E} - I_C R_2}{R_1 + R_2}, \\ \mathcal{E} - Q/C - (\dfrac{\mathcal{E} - I_C R_2}{R_1 + R_2} + I_C)R_2 = 0. \end{cases}$$

   from which one finds

$$I_C = \frac{R_1 + R_2}{R_1 R_2 C}\left(\mathcal{E}\frac{R_1 C}{R_1 + R_2} - Q\right).$$

   Then, one can identify $\lambda = \frac{R_1+R_2}{R_1 R_2}$ and thus $\tau = \frac{R_1 R_2}{R_1+R_2}$.

General Circuits	Equivalent Resistance	Exponential Decay
$\Delta V_{\text{loop}} = 0,\ I_{\text{in}} = I_{\text{out}},\ \Delta V_{\text{battery}} = \mathcal{E}$	$R_{\text{series}} = R_1 + R_2 + \cdots$	$\frac{dy}{dt} = \lambda(y_{\text{eq}} - y)$
$\Delta V_{\text{resistor}} = -IR,\ \Delta V_{\text{capacitor}} = Q/C$	$R_{\text{parallel}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$	$\implies y = (y_0 - y_{\text{eq}})e^{-\lambda t} + y_{\text{eq}}$
$P = \lvert I\Delta V\rvert = (\Delta V)^2/R = I^2 R$		$\tau = t_{1/e} = 1/\lambda = RC,\ t_{1/2} = \ln(2)t_{1/e}$

2. *Using difference between equilibrium values and the initial rate of change* Recall from lecture, albeit I told you that you should not need it, that one can deduce

$$\lambda = \frac{\text{initial rate of change}}{\text{initial difference from equilibrium}}.$$

Using what we computed for parts (a) and (b), one finds that

$$\lambda = \frac{I_0}{Q_{\text{eq}}} \frac{\mathcal{E}/R_2}{\mathcal{E}\frac{R_1 C}{R_1+R_2}} = \frac{R_1+R_2}{R_1 R_2 C} \implies \tau = \frac{R_1 R_2}{R_1+R_2} C.$$

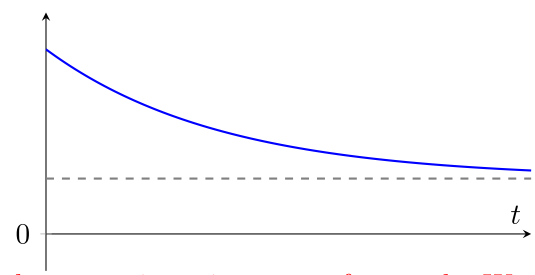
3. *Ignoring the battery, since it only changes equilibrium value*

In Mock Quiz 6(e), one found that the strength of the pump did not affect the time constant of the exponential decay in the fluid system. Similarly, the battery does not affect the time constant for exponential decay in a circuit. (This is why the time constant for charging/discharging circuits is the same!)

If one ignores the battery in the circuit, one can identify that R_1 and R_2 are in parallel relative to the capacitor, and thus $R_{\text{eq}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$ is the correct resistance to use in the $\tau = RC$ formula.

(d) For the graph below, the horizontal axis denotes time, with the capacitor discharged at $t = 0$. Which of the following quantities could be the ones plotted on the vertical axis?

- ☐ $|\Delta V_{\text{battery}}|$ ☒ $|I_{\text{battery}}|$
- ☐ $|\Delta V_{R_1}|$ ☐ $|I_{R_1}|$
- ☒ $|\Delta V_{R_2}|$ ☒ $|I_{R_2}|$
- ☐ $|\Delta V_{\text{capacitor}}|$ ☐ $|I_{\text{capacitor}}|$



This is again a question about initial and equilibrium values, rewritten in terms of a graph. We are looking for quantities that decrease their magnitude, but do not go all the way to zero.

The battery's voltage is constant.
Resistor 1's voltage starts at zero.
Resistor 2's voltage fits the description.
The capacitor's voltage starts at zero (matching resistor 1 as they are in parallel).

The battery's current fits the description.
Resistor 1's current starts at zero.
Resistor 2's current fits the descrpition.
The capacitor's current decays to zero.

General Circuits	**Equivalent Resistance**	**Exponential Decay**
$\Delta V_{\text{loop}} = 0,\ I_{\text{in}} = I_{\text{out}},\ \Delta V_{\text{battery}} = \mathcal{E}$ $\Delta V_{\text{resistor}} = -IR,\ \Delta V_{\text{capacitor}} = Q/C$ $P = \lvert I\Delta V\rvert = (\Delta V)^2/R = I^2 R$	$R_{\text{series}} = R_1 + R_2 + \cdots$ $R_{\text{parallel}} = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\cdots}$	$\frac{dy}{dt} = \lambda(y_{\text{eq}} - y)$ $\implies y = (y_0 - y_{\text{eq}})e^{-\lambda t} + y_{\text{eq}}$ $\tau = t_{1/e} = 1/\lambda = RC,\ t_{1/2} = \ln(2) t_{1/e}$