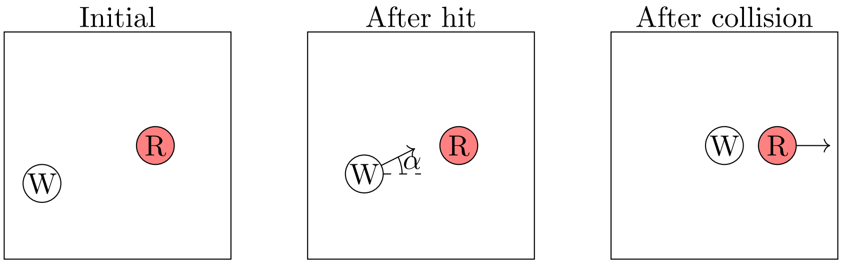
Name:

Student ID:

Problem 1:

On a frictionless pool table, a white ball (W) and a red ball (R) of equal mass are initially stationary. A player hits the white ball such that it moves at an angle α relative to the horizontal. The white ball collides with the red ball such that the red ball moves horizontally and the white ball continues to move in an unspecified direction.



(a) Which of the following are *possible* directions for the white ball's velocity after the collision? Select all that apply.

- ☒ Directly up ($v_x = 0; v_y > 0$)
- ☒ Up and to the right ($v_x > 0; v_y > 0$)
- ☒ Directly right ($v_x > 0; v_y = 0$)
- ☐ Down and to the right ($v_x > 0; v_y < 0$)
- ☐ Directly down ($v_x = 0; v_y < 0$)
- ☐ Down and to the left ($v_x < 0; v_y < 0$)
- ☐ Directly left ($v_x < 0; v_y = 0$)
- ☐ Up and to the left ($v_x < 0; v_y > 0$)

In writing the question, I had intended for $\alpha > 0$, as the white ball is pictured to be to the bottom right of the red ball. (So the exact *quantitative* description of α was not given, but the *qualitative* description is obvious from the figure.) However, in grading the results, it appears a substantial number of students included $\alpha = 0$ as a possibility. I chose not to penalize such students, allowing the case where the motion was entirely horizontal.

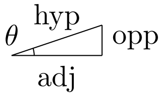
Regardless of the angle, one can make the following observation: the white ball *cannot* have $v_{f,x,\text{white}} < 0$. If it did, then by momentum conservation one would have $v_{f,x,\text{red}} > v_{i,x,\text{white}}$ and so kinetic energy would have increased:

$$\begin{aligned} 2mKE_f = |v_{f,\text{red}}|^2 + |v_{f,\text{white}}|^2 &= v_{f,x,\text{red}}^2 + v_{f,x,\text{white}}^2 + v_{f,y,\text{white}}^2 \\ &> v_{i,x,\text{white}}^2 + v_{i,y,\text{white}}^2 = |v_{i,\text{white}}|^2 = 2mKE_i. \end{aligned}$$

But, the kinetic energy cannot increase during such a process! So, $v_x > 0$ is necessary. (This argument to do with the kinetic energy is hard to notice, so do not feel bad if you did not catch it... Students receive partial credit even if they selected some options moving left, and the rest of the subtlety comes out in the curve.)

So, one can exclude all answer choices where the final velocity includes a left component.

Of the remaining options, the only ones possible are those that have $v_y > 0$ (for $\alpha > 0$) or $v_y = 0$ (for $\alpha = 0$), as necessitated by the conservation of the vertical of the momentum.

Force and Momentum	**Trigonometry**
$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$ $\vec{J} = \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$ $\lvert F_g\rvert = mg$, $g = 10$ m/s^2	$\sin(\theta) = \text{opp}/\text{hyp}$, $\cos(\theta) = \text{adj}/\text{hyp}$ $\tan(\theta) = \text{opp}/\text{adj}$, $\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$ 

(b) Select the true statement:

○ Suppose the player's hit force and time do not change; using a heavier white ball would make it move faster.
○ Suppose the white ball missed the red ball instead of hitting it; then the momentum would not be conserved.
○ It is possible for the angle α to be 90° without contradicting the rest of the description in the problem start.
⊗ It is possible for the collision to be elastic without contradicting the rest of the description in the problem start.
○ It is possible for the white ball's initial speed to be the same as the red ball's final speed.

Let us address the options in the order given:

- *Suppose the player's hit force and time do not change; using a heavier white ball would make it move faster.* If the hit force and time do not change, then $|\vec{F}|\Delta t = m|\Delta\vec{v}| = m|v_{\text{white after hit}}|$ remains the same. Increasing the mass would require the ball to move slower, not faster.

- *Suppose the white ball missed the red ball instead of hitting it; then the momentum would not be conserved.* If the collision did not happen, then the momentum would still be conserved. There are still no external forces.

- *It is possible for the angle α to be 90° without contradicting the rest of the description in the problem start.* If the initial angle was 90°, then white ball would be moving vertically prior to the collision. There would then be no horizontal momentum to transfer to the red ball. One might suppose that the red ball could still move right, and the white ball then moves left, but that is forbidden by energy conservation as described in part (a).

- *It is possible for the collision to be elastic without contradicting the rest of the description in the problem start.* There is no problem with this statement. Indeed, this is the case if the white ball's final velocity is completely vertical, since then

$$KE_f = \frac{1}{2}m\left(v_{f,x,\text{red}}^2 + v_{f,y,\text{white}}^2\right) = \frac{1}{2}m\left(v_{i,x,\text{white}}^2 + v_{i,y,\text{white}}^2\right) = KE_i$$

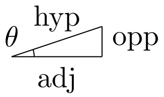
- *It is possible for the white ball's initial speed to be the same as the red ball's final speed.* If this is the case, then all of the energy must have gone to the red ball. However, we are told in the problem start that the 'white ball continues to move', so that is not possible.

(c) Suppose the player's hit force has magnitude $|\vec{F}|$ and is applied for a time Δt, and the mass of each ball is m. If the white ball moves vertically after the collision, what is the speed of each ball after the collision?

White ball : ○ $|\vec{F}|\Delta t\sin(\alpha)$ ○ $|\vec{F}|\Delta t\cos(\alpha)$ ○ $\frac{|\vec{F}|\Delta t}{m}$ ⊗ $\frac{|\vec{F}|\Delta t\sin(\alpha)}{m}$ ○ $\frac{|\vec{F}|\Delta t\cos(\alpha)}{m}$ ○ $\frac{|\vec{F}|\sin(\alpha)}{m}$ ○ $\frac{|\vec{F}|\cos(\alpha)}{m}$

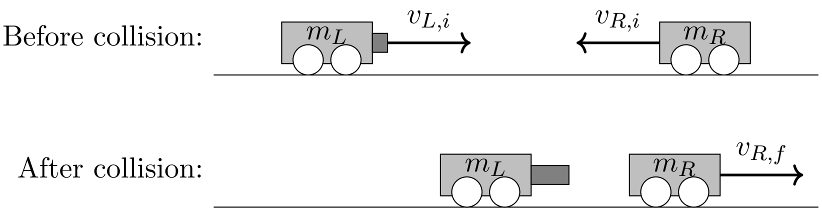
Red ball : ○ $|\vec{F}|\Delta t\sin(\alpha)$ ○ $|\vec{F}|\Delta t\cos(\alpha)$ ○ $\frac{|\vec{F}|\Delta t}{m}$ ○ $\frac{|\vec{F}|\Delta t\sin(\alpha)}{m}$ ⊗ $\frac{|\vec{F}|\Delta t\cos(\alpha)}{m}$ ○ $\frac{|\vec{F}|\sin(\alpha)}{m}$ ○ $\frac{|\vec{F}|\cos(\alpha)}{m}$

This is a direct application of $\vec{F}\Delta t = m\Delta\vec{v}$ to find the total momentum of the system of the two balls, and then some trigonometry to figure out the y-component (for the white ball) and x-component (for the red ball). Common errors included swapping sin and cos, for which I gave partial credit (as it indicates a math rather than physics error), as well as forgetting m or Δt (for which I gave lesser partial credit, as the units are strictly wrong but the idea was correct).

Force and Momentum	**Trigonometry**
$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$ $\vec{J} = \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$ $\lvert F_g\rvert = mg$, $g = 10$ m/s^2	$\sin(\theta) = \text{opp/hyp}$, $\cos(\theta) = \text{adj/hyp}$ $\tan(\theta) = \text{opp/adj}$, $\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$ 

Problem 2:

Two carts are set up to collide on a track. Initially, the left cart of mass m_L is moving with a speed $v_{L,i}$ to the right, and the right cart of mass m_R is moving with a speed $v_{R,i}$ to the left. The left cart is equipped with a spring-loaded plunger that triggers when the carts collide; the effect is that the total kinetic energy actually increases after the collision, i.e. $\Delta KE_{\text{tot}} > 0$. After the collision, the left cart ends up stationary, and the right cart moves away with a speed $v_{R,f}$.



(a)

For each of the following pairs of quantities determine their relationship, or indicate it Cannot Be Determined (CBD).

- ○ $v_{L,i} > v_{R,i}$ ○ $v_{L,i} = v_{R,i}$ ○ $v_{L,i} < v_{R,i}$ ⊗ CBD
- ○ $v_{R,i} > v_{R,f}$ ○ $v_{R,i} = v_{R,f}$ ⊗ $v_{R,i} < v_{R,f}$ ○ CBD
- ⊗ $m_L v_{L,i} > m_R v_{R,i}$ ○ $m_L v_{L,i} = m_R v_{R,i}$ ○ $m_L v_{L,i} < m_R v_{R,i}$ ○ CBD
- ⊗ $m_L v_{L,i} > m_R v_{R,f}$ ○ $m_L v_{L,i} = m_R v_{R,f}$ ○ $m_L v_{L,i} < m_R v_{R,f}$ ○ CBD
- ○ $\frac{1}{2}m_L v_{L,i}^2 > \frac{1}{2}m_R(v_{R,f}^2 - v_{R,i}^2)$ ○ $\frac{1}{2}m_L v_{L,i}^2 = \frac{1}{2}m_R(v_{R,f}^2 - v_{R,i}^2)$ ⊗ $\frac{1}{2}m_L v_{L,i}^2 < \frac{1}{2}m_R(v_{R,f}^2 - v_{R,i}^2)$ ○ CBD

The equations describing the scenario are

$$m_L v_{L,i} - m_R v_{R,i} = m_R v_{R,f}$$

from the x-component of momentum, as well as

$$\frac{1}{2}m_L v_{L,i}^2 + \frac{1}{2}m_R v_{R,i}^2 < \frac{1}{2}m_R v_{R,f}^2$$

from kinetic energy increasing.

So, let us analyze the inequalities one at a time

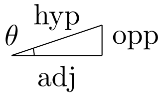
- $v_{L,i}$ and $v_{R,i}$ cannot be directly compared without the masses. It is possible for $v_{L,i} > v_{R,i}$ as in part (b), but it is also possible for $v_{R,i}$ as long as the right cart is sufficiently light. For example, consider $m_L = 10m_R = 1\text{kg}$ and $v_R = 2v_L = 1\text{m/s}$.

- $v_{R,i} < v_{R,f}$ is a consequence of the sign of the change in kinetic energy. The final velocity of the right cart needs to be larger than the initial one for the kinetic energy to go up.

- $m_L v_{L,i} > m_R v_{R,i}$ needs to be true such that the total momentum is to the right, as is clear from the final state.

- $m_L v_{L,i} > m_R v_{R,f}$ is a consequence of momentum conservation equation. Since the right cart's initial momentum is to the left, the left cart's initial momentum must be larger than the total momentum to compensate. The right cart's final momentum *is* the total momentum, so the left cart has a larger momentum than that.

- $\frac{1}{2}m_L v_{L,i}^2 < \frac{1}{2}m_R(v_{R,f}^2 - v_{R,i}^2)$ is just a rewriting of the kinetic energy increasing.

Force and Momentum	**Trigonometry**
$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$	$\sin(\theta) = \text{opp/hyp}$, $\cos(\theta) = \text{adj/hyp}$
$\vec{J} = \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$	$\tan(\theta) = \text{opp/adj}$, $\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$
$\lvert F_g\rvert = mg$, $g = 10$ m/s^2	

(b) With $m_L = 2m_R = 1.5\text{kg}$, and $v_{L,i} = 3v_{R,i} = 2.5\text{m/s}$, determine $v_{R,f}$ and ΔKE_{tot}, (Round to 2 decimal places)

$v_{R,f} =$ $\boxed{4.17}$ m/s

$\Delta KE_{\text{tot}} =$ $\boxed{1.56}$ J

Here, one can just directly solve the equations. One finds that

$$v_{R,f} = \frac{m_L v_{L,i} - m_R v_{R,i}}{m_R} = \frac{m_L v_{L,i} - \frac{1}{6} m_L v_{L,i}}{\frac{1}{2} m_L} = \frac{5}{3} v_{L,i},$$

and

$$\begin{aligned}\Delta KE = KE_f - KE_i &= \frac{1}{2} m_R v_{R,f}^2 - (\frac{1}{2} m_L v_{L,i}^2 + \frac{1}{2} m_L v_{L,f}^2) \\ &= \frac{1}{2}(\frac{m_L}{2})(\frac{5v_{L,i}}{3})^2 - \frac{1}{2} m_L v_{L,i}^2 - \frac{1}{2}(\frac{m_L}{2})(\frac{v_{L,i}}{3})^2 \\ &= \left(\frac{25}{36} - \frac{1}{2} - \frac{1}{36}\right) m_L v_{L,i}^2 = \frac{1}{6} m_L v_{L,i}^2.\end{aligned}$$

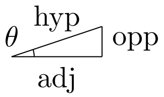
(This is actually how I solve the question, and how I recommend you do as well. It is a lot easier to rearrange a few fractions than to have 6 different numbers to remember to plug in...)

Common errors included having a + in the numerator for $v_{R,f}$, as well as computing KE_f instead of ΔKE. Both were awarded a small amount of partial credit.

(c) Imagine the experiment is set up with the exact same initial speeds, but with the piston already triggered. Without the piston's extra force, the magnitude of $|\vec{F}_{\text{by } L \text{ on } R} \Delta t|$ is reduced.
Which way do the carts end up moving after the collision in this setup?

○ L moves left and R moves right
○ Both move left
⊗ Both move right
○ L moves right and R moves left
○ Cannot be determined

With the piston's force removed, the right cart experiences a lesser impulse to the right. Consequently, the left cart experiences a lesser impulse to the left. Since the left cart's change is smaller than necessary to stop it, it must end up moving right. Since the left cart ends up moving right, the right cart must've still turned around. So, both carts move to the right.

<table>
<tr><th>Force and Momentum</th><th>Trigonometry</th></tr>
<tr><td>$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$
$\vec{J} = \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$
$|F_g| = mg$, $g = 10\ \text{m/s}^2$</td><td>$\sin(\theta) = \text{opp/hyp}$, $\cos(\theta) = \text{adj/hyp}$
$\tan(\theta) = \text{opp/adj}$, $\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$
 </td></tr>
</table>