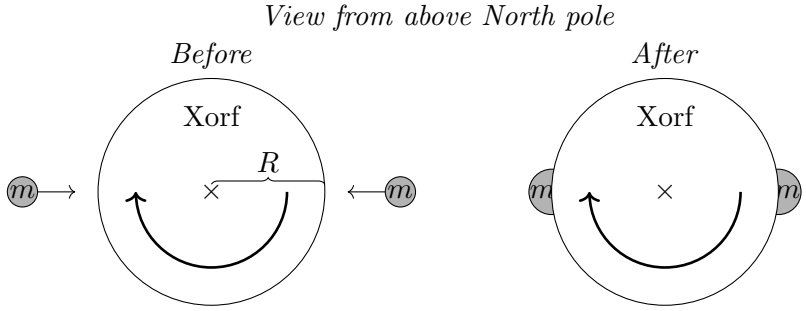
Quiz 4 (compiled : 2026/05/25 at 08:21; version 1)

Name: **Student ID:**

Problem 1:

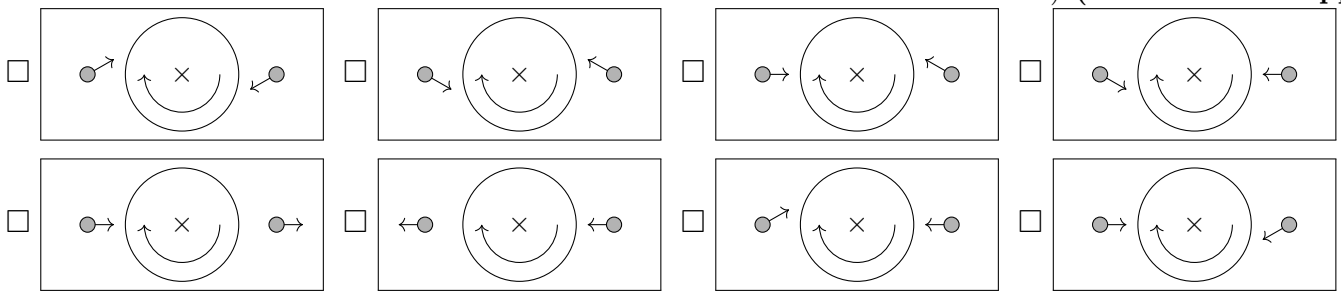
Planet Xorf (of mass M_X and radius R) is rotating around its vertical axis, such that the rotation looks clockwise from the North pole. Two small asteroids (of equal mass m) hit Xorf's equator from opposite sides at equal speeds. The asteroids' velocities were directed towards the planet's axis. The asteroids stick to the planet and rotate with it. Ignore the effects of gravity. (The moment of inertia of a sphere around its axis is $I_{\text{sphere}} = \frac{2}{5} m_{\text{sphere}} (r_{\text{sphere}})^2$.)



(a) Suppose $m = M_X/10$, so the asteroids are each one-tenth of the mass of Xorf. If one Xorfian day before the catastrophe was 12 hours, how long is it afterwards?
(One day is the amount of time it takes for a planet to complete one full rotation.)

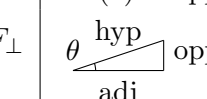
◯ 4 hours ◯ 6 hours ◯ 8 hours ◯ 10 hours ◯ 12 hours ◯ 15 hours ◯ 18 hours ◯ 20 hours

(b) A Xorfian mission to redirect the asteroids is sent out with the following objective: ensure that the angular speed of the planet remains the same after the collision. Which of the following potential redirections could work? (The arrows on the asteroids denote the direction of their velocities after the mission.) **(Select all that apply.)**



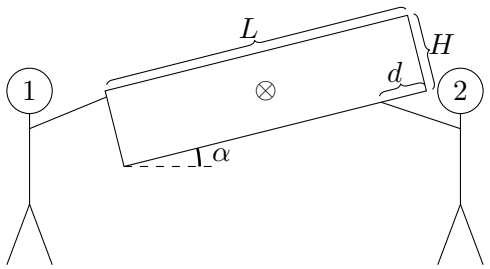
(c) For each of the following vectors, identify the direction it points:

- Xorf's initial angular speed ($\vec{\omega}_{X,i}$) : ◯ right ◯ left ◯ up ◯ down ◯ out of page ◯ into page
- Left asteroid's initial velocity ($\vec{v}_{l,i}$) : ◯ right ◯ left ◯ up ◯ down ◯ out of page ◯ into page
- Torque onto Xorf during the collision ($\vec{\tau}_{\text{on } X}$) : ◯ right ◯ left ◯ up ◯ down ◯ out of page ◯ into page
- Right asteroid's angular momentum change ($\Delta\vec{L}_r$) : ◯ right ◯ left ◯ up ◯ down ◯ out of page ◯ into page

Force and Momentum	**Angular Quantities**	**Trigonometry**
$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$ $\vec{J} = \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$	$\vec{L} = I\vec{\omega}$, $I_{\text{point}} = mr^2$ $\vec{\tau}\Delta t = I\Delta\vec{\omega} = \Delta\vec{L}$	$\sin(\theta) = \text{opp/hyp}$, $\cos(\theta) = \text{adj/hyp}$ $\tan(\theta) = \text{opp/adj}$, $\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$
$\lvert F_g\rvert = mg$, $g = 10$ m/s^2	$\lvert\tau\rvert = \lvert F\rvert\lvert r\rvert\sin(\theta) = \lvert F\rvert r_\perp = \lvert r\rvert F_\perp$ $\lvert L_{\text{linear}}\rvert = \lvert p\rvert\lvert r\rvert\sin(\theta) = \lvert p\rvert r_\perp$	 1 rot = 360 deg = 2π rad

Problem 2:

A fridge (i.e. a uniform rectangular block of dimensions L by H) is held at an angle α by two people. One person is pushing on the shorter side at the top-left corner. The second person is pushing on the longer side a distance d from the bottom-right corner. The fridge has a mass M, and is sufficiently slippery that friction is to be ignored; the two people are applying only normal forces.



For all parts, suppose the fridge is stationary. Gravity acts on the center of the fridge (⊗).

(a) What is the ratio of the magnitudes of the two persons' forces $|F_1|/|F_2|$? *(Hint: force balancing is useful here.)*

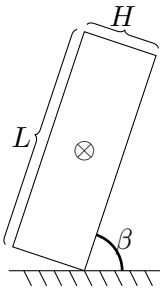
◯ 1 ◯ $\sin(\alpha)$ ◯ $\cos(\alpha)$ ◯ $\sin(\alpha)/\cos(\alpha)$ ◯ $\cos(\alpha)/\sin(\alpha)$ ◯ $\sin(90^\circ+\alpha)$ ◯ $\cos(90^\circ+\alpha)$

(b) Suppose one has $|F_1| = |F_2|$. What must the distance d be? *(Hint: torque balancing is useful here.)*

◯ 0 ◯ L ◯ $L-H$ ◯ $H-L$ ◯ H ◯ $L/2$ ◯ $(L-H)/2$ ◯ $(H-L)/2$ ◯ $H/2$

(c) Suppose $d = 0$. What is the ratio of the two persons' forces $|F_1|/|F_2|$? *(Hint: torque balancing is useful here.)*
◯ 1 ◯ H/L ◯ L/H ◯ $2H/L$ ◯ $2L/H$ ◯ $H/(2L)$ ◯ $L/(2H)$

(d) The two people balance the fridge on a corner as shown. The fridge remains stationary without the people pushing on it. What is the angle β?



(Math hint: $\sin(\theta) = a/b \leftrightarrow \arcsin(a/b) = \theta$,
$\cos(\theta) = a/b \leftrightarrow \arccos(a/b) = \theta$,
$\tan(\theta) = a/b \leftrightarrow \arctan(a/b) = \theta$.*)*
(Each answer choice is written in two ways for your convenience.)

◯ $\arcsin(H/L) = \arccos(L/H)$ ◯ $\arcsin(L/H) = \arccos(H/L)$

◯ $\arcsin(\frac{2H}{L}) = \arccos(\frac{L}{2H})$ ◯ $\arcsin(\frac{2L}{H}) = \arccos(\frac{H}{2L})$

◯ $\arctan(H/L) = 90^\circ - \arctan(L/H)$ ◯ $\arctan(L/H) = 90^\circ - \arctan(H/L)$

◯ $\arctan(\frac{2H}{L}) = 90^\circ - \arctan(\frac{L}{2H})$ ◯ $\arctan(\frac{2L}{H}) = 90^\circ - \arctan(\frac{H}{2L})$

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$\lvert F_g\rvert = mg$, $g = 10$ m/s^2	$\lvert\tau\rvert = \lvert F\rvert\lvert r\rvert\sin(\theta) = \lvert F\rvert r_\perp = \lvert r\rvert F_\perp$ $\lvert L_{\text{linear}}\rvert = \lvert p\rvert\lvert r\rvert\sin(\theta) = \lvert p\rvert r_\perp$	θ hyp / adj, opp 1 rot = 360 deg = 2π rad