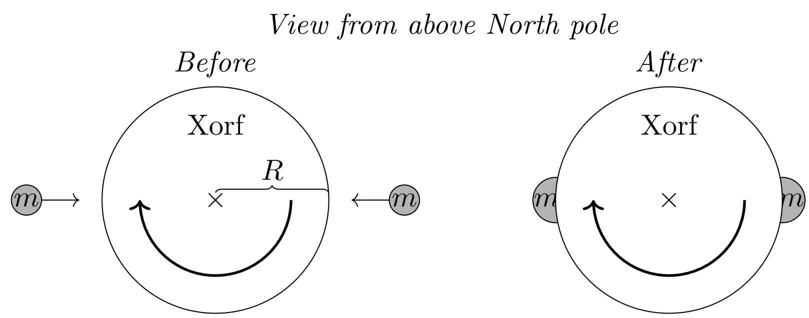
Quiz 4 (compiled : 2026/05/27 at 22:16; version 1)

Name: ______________________ **Student ID:** ______________________

Problem 1:

Planet Xorf (of mass M_X and radius R) is rotating around its vertical axis, such that the rotation looks clockwise from the North pole. Two small asteroids (of equal mass m) hit Xorf's equator from opposite sides at equal speeds. The asteroids' velocities were directed towards the planet's axis. The asteroids stick to the planet and rotate with it. Ignore the effects of gravity. (The moment of inertia of a sphere around its axis is $I_{\text{sphere}} = \frac{2}{5} m_{\text{sphere}} (r_{\text{sphere}})^2$.)



Before getting into the problem itself, let's analyze what could be gleaned from the problem start.

1. This is remarkably similar to the DL15 mock quiz. There is a rotating object, to which extra masses are about to be attached.
2. The moment of inertia is about to go up, so we expect the angular speed to go down. [See also Activity 7.13(A) where this was found for rotating on a stool and pulling your arms in, as well as 'Lecture 7 - Lecturing' slide 25.]
3. The objects are directed such that $r_\perp$ (i.e. the component of the vector $\vec{r}$ from the axis of rotation to their location that is perpendicular to the momentum) is zero. Consequently, their angular momentum is zero from the formula $|L_{\text{linear}} = |p| r_\perp$. This makes sense, since they are not going around the axis one way or the other.
4. Nonetheless, given the similarity to the mock quiz, one should expect that a potential modification would be that the asteroids would start with some angular momentum instead.
5. Naturally, given the emphasis given to the right-hand-rule in lecture and in DL, one should anticipate calculating the direction of some angular quantities.

With this in mind, every part is something that one could anticipate more or less just from the problem start. Part (a) uses the change in moment of inertia. Part (b) asks about the angular momentum caused by linear motion. Part (c) asks one to use the right hand rule for each angular quantity one can consider: angular speed, torque, and angular momentum.

Force and Momentum	**Angular Quantities**	**Trigonometry**
$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$	$\vec{L} = I\vec{\omega}$, $I_{\text{point}} = mr^2$	$\sin(\theta) = \text{opp}/\text{hyp}$, $\cos(\theta) = \text{adj}/\text{hyp}$
$\vec{J} = \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$	$\vec{\tau}\Delta t = I\Delta\vec{\omega} = \Delta\vec{L}$	$\tan(\theta) = \text{opp}/\text{adj}$, $\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$
$\lvert F_g\rvert = mg$, $g = 10\ \text{m/s}^2$	$\lvert\tau\rvert = \lvert F\rvert\lvert r\rvert\sin(\theta) = \lvert F\rvert r_\perp = \lvert r\rvert F_\perp$ $\lvert L_{\text{linear}}\rvert = \lvert p\rvert\lvert r\rvert\sin(\theta) = \lvert p\rvert r_\perp$	θ hyp opp adj 1 rot = 360 deg = 2π rad

(a) Suppose $m = M_X/10$, so the asteroids are each one-tenth of the mass of Xorf. If one Xorfian day before the catastrophe was 12 hours, how long is it afterwards?
(One day is the amount of time it takes for a planet to complete one full rotation.)

○ 4 hours ○ 6 hours ○ 8 hours ○ 10 hours ○ 12 hours ○ 15 hours ⊗ 18 hours ○ 20 hours

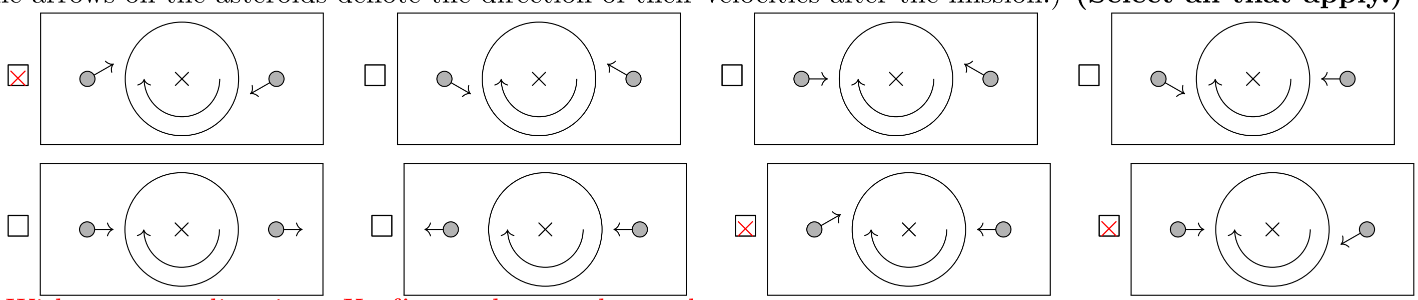
The moment of inertia before the collision was $\frac{2}{5}M_X R^2$, as given in the problem start. The moment of inertia of the additional asteroids is $2 \times (\frac{M_X}{10}R^2)$, since there are two (2) asteroids each of mass $\frac{M_X}{10}$ at a distance R from the axis of rotation. Even without a calculator, one can identify that $\frac{2}{10} + \frac{2}{5} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5}$, so the final moment of inertia is $\frac{3}{2}$ of the initial moment of inertia.

Consequently, due to the conservation of angular momentum, $I_i\omega_i = I_f\omega_f$, so ω_f must be two thirds of ω_i. (The planet spins more slowly 'due to the extra weight'.)

If the planet spins more slowly, the days must end up being longer. Since it is spinning two thirds as slowly, the days are exactly three halves as long, so $\frac{3}{2} \times 12$ hours in this version (and $\frac{3}{2} \times 36$ hours in the other version).

The most common mistake was considering only a single asteroid (which leads to the ratio $\frac{6}{5}$ instead)... Putting both the correct solutions and the single-asteroid solutions together, still only ∼50% found the correct answer.

(b) A Xorfian mission to redirect the asteroids is sent out with the following objective: ensure that the angular speed of the planet remains the same after the collision. Which of the following potential redirections could work? (The arrows on the asteroids denote the direction of their velocities after the mission.) **(Select all that apply.)**



Without any redirections, Xorf's angular speed goes down.

So, whatever the asteroids are redirected to do should increase the total angular momentum of the system. This is possible only if the asteroids are redirected to match the direction Xorf is rotating, i.e. they also go clockwise around the axis of rotation. (The remaining solutions either make things worse by slowing the planet down more, or have only a single asteroid hitting, which is still a problem.)

I thought this would be a more approachable question than asking for explicit computations. Nonetheless, the average on this question was still ∼40%.

(c) For each of the following vectors, identify the direction it points:

- Xorf's initial angular speed ($\vec{\omega}_{X,i}$) : ○ right ○ left ○ up ○ down ○ out of page ⊗ into page
- Left asteroid's initial velocity ($\vec{v}_{l,i}$) : ⊗ right ○ left ○ up ○ down ○ out of page ○ into page
- Torque onto Xorf during the collision ($\vec{\tau}_{\text{on } X}$) : ○ right ○ left ○ up ○ down ⊗ out of page ○ into page
- Right asteroid's angular momentum change ($\Delta\vec{L}_r$) : ○ right ○ left ○ up ○ down ○ out of page ⊗ into page

The initial angular speed is clockwise, which with the right-hand-rule one finds is into the page. The same is true for the angular momentum change of either asteroid.

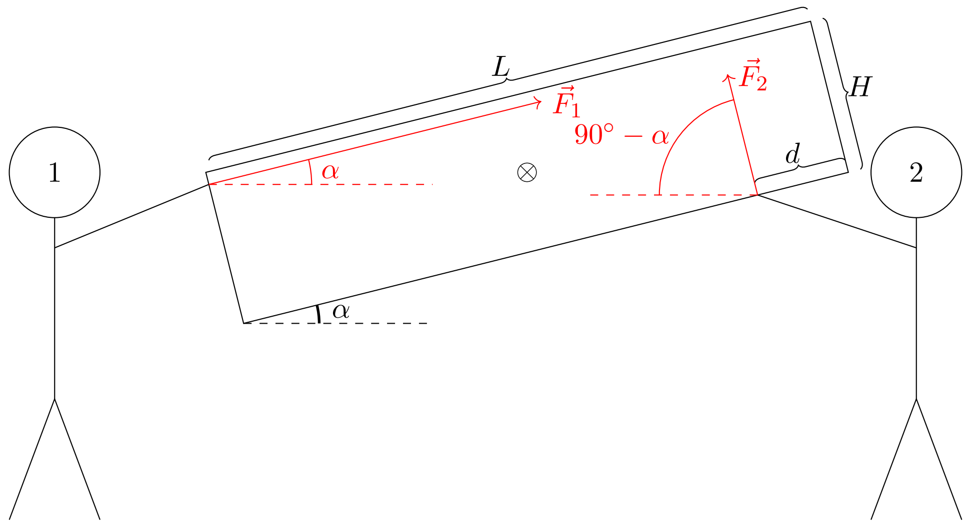
Since Xorf's rotation slows down, the torque on Xorf must be in the opposite direction, i.e. out of the page. (This was the part with the most wrong answers, as ∼50% of students still put into the page.)

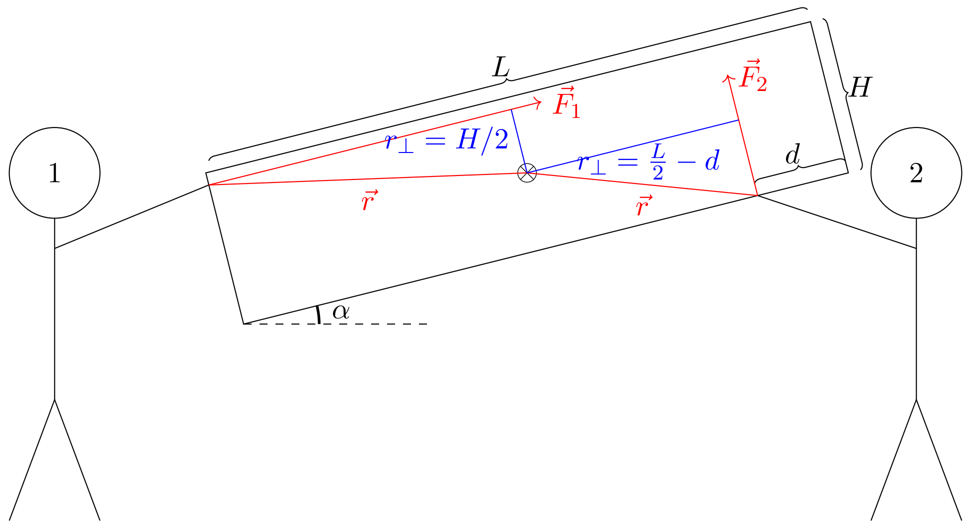
I added the asteroid's initial velocity so it would make sense to include the right/left/up/down options for some variety. This velocity is labeled directly in the picture for the problem start; the left asteroid is moving right and the right asteroid is moving left.

Force and Momentum	**Angular Quantities**	**Trigonometry**
$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$	$\vec{L} = I\vec{\omega}$, $I_{\text{point}} = mr^2$	$\sin(\theta) = \text{opp/hyp}$, $\cos(\theta) = \text{adj/hyp}$
$\vec{J} = \vec{F}\Delta t = m\Delta\vec{v} = \Delta\vec{p}$	$\vec{\tau}\Delta t = I\Delta\vec{\omega} = \Delta\vec{L}$	$\tan(\theta) = \text{opp/adj}$, $\text{adj}^2 + \text{opp}^2 = \text{hyp}^2$
$\lvert F_g\rvert = mg$, $g = 10\ \text{m/s}^2$	$\lvert\tau\rvert = \lvert F\rvert\lvert r\rvert\sin(\theta) = \lvert F\rvert r_\perp = \lvert r\rvert F_\perp$ $\lvert L_{\text{linear}}\rvert = \lvert p\rvert\lvert r\rvert\sin(\theta) = \lvert p\rvert r_\perp$	θ hyp opp adj 1 rot = 360 deg = 2π rad

Problem 2:

A fridge (i.e. a uniform rectangular block of dimensions L by H) is held at an angle α by two people. One person is pushing on the shorter side at the top-left corner. The second person is pushing on the longer side a distance d from the bottom-right corner. The fridge has a mass M, and is sufficiently slippery that friction is to be ignored; the two people are applying only normal forces.





For all parts, suppose the fridge is stationary. Gravity acts on the center of the fridge (⊗).

The most complete set up to this problem would write down all the equations to balance forces and torques. With the axis of rotation chosen to be the center of mass, one finds that

$$F_{x,\text{net}} = |F_1|\cos(\alpha) - |F_2|\sin(\alpha) = 0,$$
$$F_{y,\text{net}} = |F_1|\sin(\alpha) + |F_2|\cos(\alpha) - Mg = 0,$$
$$\tau_{\text{out of page,net}} = |F_2|\left(\frac{L}{2} - d\right) - |F_1|\frac{H}{2} = 0,$$

where the easiest formula to use for torque is $|F|r_\perp$. Before coming to the quiz, one should have definitely known how to set this up!

Nonetheless, since it is a bit unwieldy, I structured each part to depend on only one equation at a time.

Force and Momentum	**Angular Quantities**	**Trigonometry**
$\vec{p} = m\vec{v}$, $KE = \frac{1}{2}mv^2$	$\vec{L} = I\vec{\omega}$, $I_{\text{point}} = mr^2$	$\sin(\theta) = \text{opp/hyp}$, $\cos(\theta) = \text{adj/hyp}$
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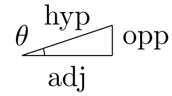
$\lvert F_g\rvert = mg$, $g = 10$ m/s^2	$\lvert\tau\rvert = \lvert F\rvert\lvert r\rvert\sin(\theta) = \lvert F\rvert r_\perp = \lvert r\rvert F_\perp$ $\lvert L_{\text{linear}}\rvert = \lvert p\rvert\lvert r\rvert\sin(\theta) = \lvert p\rvert r_\perp$	 1 rot = 360 deg = 2π rad

(a) What is the ratio of the magnitudes of the two persons' forces $|F_1|/|F_2|$? *(Hint: force balancing is useful here.)*

○ 1 ○ $\sin(\alpha)$ ○ $\cos(\alpha)$ ⊗ $\sin(\alpha)/\cos(\alpha)$ ○ $\cos(\alpha)/\sin(\alpha)$ ○ $\sin(90° + \alpha)$ ○ $\cos(90° + \alpha)$

Using the first equation,

$$|F_1|\cos(\alpha) - |F_2|\sin(\alpha) = 0 \implies \frac{|F_1|}{|F_2|} = \frac{\sin(\alpha)}{\cos(\alpha)}.$$

In words: since gravity has no horizontal component, the two people must balance the horizontal components of their forces. Since the horizontal component of the first person depends on the cosine of the fridge's angle, and the horizontal component of the second person depends on the sine of the fridge's angle, the ratio of their forces must be the ratio of these trig functions. (If one is uncertain which way the ratio goes, one can consider a specific angle. For example, for $\alpha = 0$, the second person no longer applies any horizontal force, so the first person must not push at all! So, for $\alpha = 0$, $|F_1|/|F_2|$ is zero. This is true only if $\sin(\alpha)$ is in the numerator.)

(b) Suppose one has $|F_1| = |F_2|$. What must the distance d be? *(Hint: torque balancing is useful here.)*

○ 0 ○ L ○ $L - H$ ○ $H - L$ ○ H ○ $L/2$ ⊗ $(L - H)/2$ ○ $(H - L)/2$ ○ $H/2$

Using the third equation,

$$|F_2|\left(\frac{L}{2} - d\right) - |F_1|\frac{H}{2} = 0 \overset{|F_1|=|F_2|}{\implies} \left(\frac{L}{2} - d\right) - \frac{H}{2} = 0 \implies d = \frac{L-H}{2}$$

In words: in order for the torques to still be balanced while the forces are equal, their level arms must be the same. In order for that to be true, the second person's distance from the corner must be just right such that the perpendicular component of their distance from the center matches the first person.

(c) Suppose $d = 0$. What is the ratio of the two persons' forces $|F_1|/|F_2|$? *(Hint: torque balancing is useful here.)*

○ 1 ○ H/L ⊗ L/H ○ $2H/L$ ○ $2L/H$ ○ $H/(2L)$ ○ $L/(2H)$

Using the third equation,

$$|F_2|\left(\frac{L}{2} - d\right) - |F_1|\frac{H}{2} = 0 \overset{d=0}{\implies} |F_2|\frac{L}{2} - |F_1|\frac{H}{2} = 0 \implies \frac{|F_1|}{|F_2|} = \frac{L}{H}$$

In words: in order for their torques to be balanced, the forces must have the opposite ratio of the perpendicular components of their distances from the axis of rotation.

(d) The two people balance the fridge on a corner as shown. The fridge remains stationary without the people pushing on it. What is the angle β?

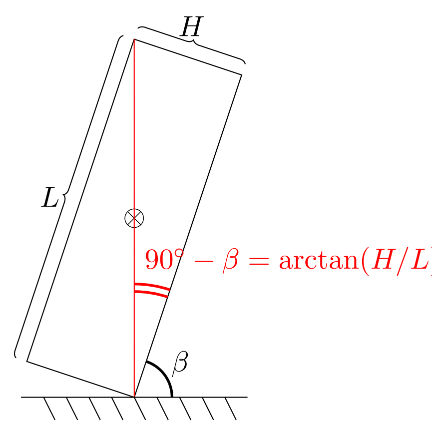
(Math hint: $\sin(\theta) = a/b \leftrightarrow \arcsin(a/b) = \theta$,
$\cos(\theta) = a/b \leftrightarrow \arccos(a/b) = \theta$,
$\tan(\theta) = a/b \leftrightarrow \arctan(a/b) = \theta$.*)*

(Each answer choice is written in two ways for your convenience.)

○ $\arcsin(H/L) = \arccos(L/H)$ ○ $\arcsin(L/H) = \arccos(H/L)$
○ $\arcsin(\frac{2H}{L}) = \arccos(\frac{L}{2H})$ ○ $\arcsin(\frac{2L}{H}) = \arccos(\frac{H}{2L})$
○ $\arctan(H/L) = 90° - \arctan(L/H)$ ⊗ $\arctan(L/H) = 90° - \arctan(H/L)$
○ $\arctan(\frac{2H}{L}) = 90° - \arctan(\frac{L}{2H})$ ○ $\arctan(\frac{2L}{H}) = 90° - \arctan(\frac{H}{2L})$

In order to be balanced, the center of the fridge must be exactly above the corner (as pictured, so I essentially did the physics step for you with the drawing).

What is left is a geometry exercise (link to Khan Academy practice).



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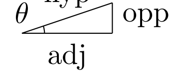
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