Winter 2025, PHY7B Practice Solutions for OH1 *TA: Artyom Lisitsyn, alisitsyn@ucdavis.edu*

Question 1 'What the faucet?'

A faucet for water is turned on, where the opening for the water is a circle 2 cm across. The water comes out with a speed of 1 m/s, and then falls 15 cm into the bottom of the sink. Assume that the cross-section of the stream of water is a circle for all heights.

1. *How fast is the water moving when it hits the bottom of the sink?*

   As the water is falling through the air, the pressure is constant (always atmospheric), and there is no pump or resistance. So, the Bernoulli equation simplifies to

   $$\frac{1}{2}\rho\Delta(v^2) + \rho g\Delta h = 0.$$

   Rearranging the equation, we can solve for the final velocity as

   $$\begin{aligned}\frac{1}{2}\rho v_f^2 = -\rho g\Delta h + \frac{1}{2}\rho v_i^2 \implies v_f &= \sqrt{-2g\Delta h + v_i^2} \\ &= \sqrt{-2\cdot 10\text{ m/s}^2 \cdot (-0.15\text{ m}) + (1\text{ m/s})^2} \\ &= 2\text{ m/s}.\end{aligned}$$

2. *What is the final width of the stream when it hits the bottom of the sink?*

   The flow of water is constant, after all no water appears or disappears as it is falling:

   $$I_{top} = I_{bottom} \implies A_{top}v_{top} = A_{bottom}v_{bottom} \implies \frac{A_{bottom}}{A_{top}} = \frac{v_{top}}{v_{bottom}}$$
   $$\implies \frac{\frac{1}{4}\pi d_{bottom}^2}{\frac{1}{4}\pi d_{top}^2} = \frac{v_{top}}{v_{bottom}} \implies d_{bottom} = \sqrt{v_{top}/v_{bottom}}\cdot d_{top}.$$

   Plugging in the numbers from the problem and the previous part,

   $$d_{bottom} = \sqrt{1/2}\cdot(2\text{ cm}) \simeq 1.41\text{ cm}.$$

3. *How does the final width change if you quadruple the initial width of the stream? (i.e. 8 cm instead of 2 cm)*

   From the previous part, we see that the width at the bottom is proportional to the width at the top. So, quadrupling the initial width will quadruple the final width, giving us approximately 5.65 cm.

4. *How does the final width change if you quadruple the height the water falls? (i.e. 60 cm instead of 15 cm)*

   Quadrupling the height that the water falls will not directly quadruple the width, because we first need to use the energy-conservation (Bernoulli) equation to understand the intermediate variables!

   We find that the final velocity is

   $$\begin{aligned}v_f &= \sqrt{-2g\Delta h + v_i^2} \\ &= \sqrt{-2\cdot 10\text{ m/s}^2\cdot(-0.6\text{ m}) + (1\text{ m/s})^2} \\ &\simeq 3.61\text{ m/s},\end{aligned}$$

   so then we find that the final width is

   $$d_{bottom} = \sqrt{1/3.61}\cdot(2\text{ cm}) \simeq 1.05\text{ cm}.$$

Question 2 'Rough but steady'

A vertical pipe with water flowing down is designed to have a constant pressure *and* constant width *throughout. Unfortunately the change in gravitational energy density may cause the pipe to have an increasing pressure near the bottom.*

1. *Explain how adding resistance to the pipe may resolve the issue.* Since there is no pump in the pipe, and the constant width ensures constant speed/kinetic energy, the Bernoulli equation is

$$\Delta P = -\frac{\Delta PE_{grav}}{V} - \frac{\Delta E_{th}}{V}.$$

Since the water is flowing down, we have that $\Delta PE_{grav} < 0$, so the change in gravitational potential energy contributes a positive term overall on the right hand side. This is consistent with the intuition and question's statement that the pressure would get higher lower in the pipe.

However, if we manage to produce thermal energy $\Delta E_{th} = -\Delta PE_{grav}$, we will cancel out this positive term and reach a constant pressure as desired.

2. *Suppose the resistance for a section of the pipe of length ℓ is $R = \tilde{R}\ell$. In terms of $\tilde{R}$ (resistance per unit length), ρ (fluid mass density), and g (gravitational acceleration), determine the flow in the pipe necessary to have a constant pressure. Verify the units match.*

Let us write down the Bernoulli equation in terms of the indicators for a section of pipe with a length ℓ. Then, we would use $\Delta PE_{grav} = \rho g(-\ell)$ (since we moved down), and $\Delta E_{th}/V = IR = I\tilde{R}\ell$. This gives us

$$\Delta P = \rho g \ell - I\tilde{R}\ell = 0 \implies I = \frac{\rho g}{\tilde{R}}.$$

To verify the units, we should first figure out the units of $\tilde{R}$:

$$[\Delta E_{th}/V] = \text{J / m}^3 = \text{kg /(m s}^2) = [I\tilde{R}\ell] = \text{m}^3\text{/s} \cdot [\tilde{R}] \cdot \text{m} \implies [\tilde{R}] = \text{kg / (m}^5\text{ s)}$$

Then we find,

$$\left[\frac{\rho g}{\tilde{R}}\right] = \text{kg / m}^3\text{m / s}^2/(\text{kg / (m}^5\text{ s)}) = \text{m}^3\text{ / s},$$

as expected for flow (volume per unit time).