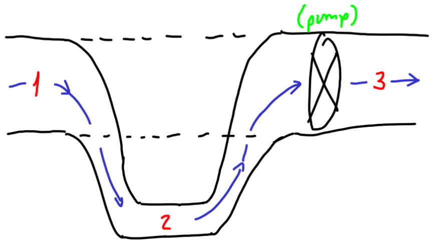
Winter 2025, PHY7B Practice Questions for OH2 *TA: Artyom Lisitsyn, alisitsyn@ucdavis.edu*

Question 1 'Ranking'

Assuming no dissipation, for each of the following (in)equalities, either choose the correct symbol, or explain why there isn't enough information:



$$v_1 \overset{>}{\underset{=}{<}} v_2, \quad v_1 \overset{>}{\underset{=}{<}} v_3, \quad v_2 \overset{>}{\underset{=}{<}} v_3$$

$$P_1 \overset{>}{\underset{=}{<}} P_2, \quad P_1 \overset{>}{\underset{=}{<}} P_3, \quad P_2 \overset{>}{\underset{=}{<}} P_3$$

To deduce the relationships between the speeds, since we have continuous flow in one pipe, we must have

$$I_1 = I_2 = I_3 \implies A_1 v_1 = A_2 v_2 = A_3 v_3.$$

Since $A_1 = A_3 > A_2$, we must have $v_1 = v_3 < v_2$.

To deduce the relationship between the pressures, we compare what types of energy got larger or smaller betwen different points.

Between points 1 and 2 we have:

$$\Delta P + \overbrace{\frac{\Delta KE}{vol}}^{(+)} + \overbrace{\frac{\Delta PE}{vol}}^{(-)} = \cancelto{0}{\frac{\Delta E_{pump}}{vol}} - \cancelto{0}{\frac{\Delta E_{th}}{vol}},$$

so we cannot deduce whether the pressure increased or decreased at it depends on the magnitudes of the KE and PE changes.

Similarly, between points 2 and 3 we have:

$$\Delta P + \overbrace{\frac{\Delta KE}{vol}}^{(-)} + \overbrace{\frac{\Delta PE}{vol}}^{(+)} = \overbrace{\frac{\Delta E_{pump}}{vol}}^{(+)} - \cancelto{0}{\frac{\Delta E_{th}}{vol}},$$

so again due to terms with different signs, we cannot deduce the change in pressure without having numbers.

However, between points 1 and 3 we have:

$$\Delta P + \cancelto{0}{\frac{\Delta KE}{vol}} + \cancelto{0}{\frac{\Delta PE}{vol}} = \overbrace{\frac{\Delta E_{pump}}{vol}}^{(+)} - \cancelto{0}{\frac{\Delta E_{th}}{vol}},$$

so $\Delta P > 0$ in this case, i.e. $P_3 > P_2$.

So, the correct signs to use in the (in)equalities above are

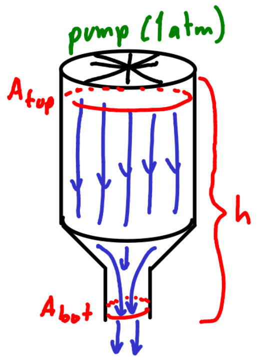
$$v_1 < v_2, \quad v_1 = v_3, \quad v_2 > v_3$$

$$P_1 ? P_2, \quad P_1 < P_3, \quad P_2 ? P_3$$

where the ones with question marks cannot be determined without having more precise information.

Question 2 'The Funnel'

A pump is pushing water at a fixed pressure of 1 *atm into a frictionless funnel. The bottom of the funnel is open to the atmosphere. The top area of the funnel is* $A_{top} = 1\ cm^2$*, and the bottom area is* $A_{bottom} = 0.5\ cm^2$*. The height of the water in the funnel* $h = 1$ *m.*



What is the speed of the water as it exits the funnel?

Setting up Bernoulli's equation between the top and bottom points

$$\Delta P + \frac{1}{2}\rho\Delta(v^2) + \rho g\Delta h = 0,$$

where we do not have a pump or friction as the water flows through the pump.

Since the water enters at a fixed pressure of 1 atm, and exits the funnel also open to the atmosphere, we actually have that $\Delta P = P_{top} - P_{bot} = (1\text{ atm} - 1\text{ atm}) = 0$.

So, our equation is

$$\frac{1}{2}\rho(v_{bot}^2 - v_{top}^2) + \rho g\Delta h = 0.$$

We know that the flow is continuous throughout the pipe, and so

$$I_{top} = I_{bot} \implies v_{top}A_{top} = v_{bot}A_{bot} \implies v_{top} = \frac{A_{bot}}{A_{top}}v_{bot}.$$

Then, our equation is

$$\frac{1}{2}\rho\left(v_{bot}^2 - \left[\frac{A_{bot}}{A_{top}}v_{bot}\right]^2\right) + \rho g\Delta h = 0$$

$$\frac{1}{2}\rho\left(1 - \frac{A_{bot}^2}{A_{top}^2}\right)v_{bot}^2 = -\rho g\Delta h$$

$$v_{bot} = \sqrt{\frac{-2g\Delta h}{1 - \frac{A_{bot}^2}{A_{top}^2}}} = \sqrt{\frac{-2\cdot(10\text{ m/s}^2\cdot(-1\text{ m}))}{1-0.5^2}} = 5.16\text{ m/s}.$$

Note that $\Delta h = -1$ m, since we are moving downwards. This cancels out the other minus sign in the square root, preventing us from taking the square root of a negative number.

One could have also solved the problem using $\Delta(v^2) = I^2\Delta(1/A^2)$, finding the flow rate first. Then, one finds the velocity as $v_{bot} = I/A_{bot}$.

Question 3 'The Standpipes'

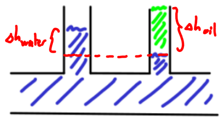
A frictionless horizontal pipe of constant width is attached to two standpipes, both of which are open to the atmosphere. In each of the following cases, circle the image that best describes what one would see once equilibrium is reached, and then calculate the height difference between the surfaces of water.

Flowing water: *Water flows to the right in the horizontal pipe with a flow rate of* $I = 1$ *m*^{3}*/s.*

Since the pipe is frictionless, the pressure at the bottom of the standpipes is actually equal. Consequently, the heights of the water columns in the standpipes are equal: $\Delta h_{water} = 0$.

Adding oil: *The right standpipe has a* 30 *cm tall layer of oil (0.9 kg/m*^{3}*) added to it.*
(There is no flow in the horizontal pipe.
Note that the height difference should be calculated between the surfaces of water, not the surface of the oil.)

On one hand, the oil pushes down on the water, so the water levels are not equal. On the other hand, since the oil is less dense, the left water column does not need to go up as high to make pressures match. Consider drawing a reference line at the level of the water in the right column.



The pressures match at the dashed red line: the points where it intersects the standpipes are at same height in the stationary water. The pressures also match at the surface of the water in the left pipe and oil in the right pipe, as both are open to the atmosphere. Therefore, we have

$$\Delta P_{left} = \Delta P_{right}$$

$$\Delta P_{left} + \rho_{water} g \Delta h_{water} = 0, \quad \Delta P_{left} + \rho_{oil} g \Delta h_{oil} = 0$$

$$\rho_{water} g \Delta h_{water} = \rho_{oil} g \Delta h_{oil} \implies \Delta h_{water} = \frac{\rho_{oil}}{\rho_{water}} \Delta h_{oil} = 0.9 \cdot 30 \text{ cm} = 27 \text{ cm}.$$

Adding piston: *The right standpipe has an airtight piston applying* 1.1 *atm of pressure.*
(There is no flow in the horizontal pipe.)

Similarly to the oil, the piston pushing down lowers the water level. Instead of considering two columns, we simply match the pressures at the level of the piston, knowing that they must both be 1.1 atm. So, Bernoulli's equation for the left standpipe gives us

$$\Delta P + \rho g \Delta h = 0 \implies (P_{atm} - P_{piston}) + \rho g \Delta h = 0$$

$$\Delta h = \frac{P_{piston} - P_{atm}}{\rho g} = \frac{0.1 \text{ atm}}{1000 \text{ kg/m}^3 \cdot 10 \text{ m/s}^2} = \frac{10^4 \text{ Pa}}{10^4 \text{ Pa/m}} = 1 \text{ m}.$$