PHY7A Practice Questions

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Fall 2024

These questions are written with the intent to be challenging applications of the material from the course. The concepts and problem-solving strategies are relevant, but are often beyond the expectations. **For direct preparation for the final, consider reviewing the DLs, and working through the past finals available on Canvas first.**

A lot of the questions are intended to give an extra challenge, and push just beyond the course. Here is an outline that roughly describes the concepts and skills:

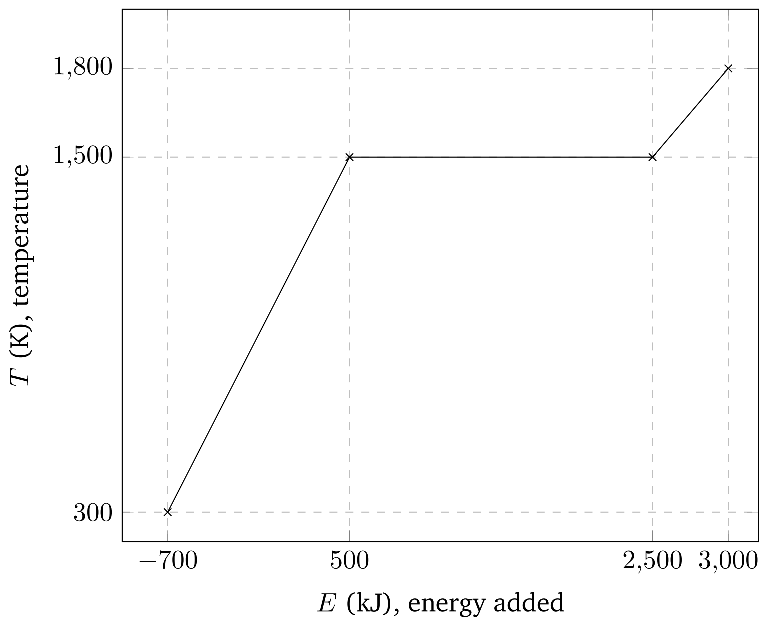
- **1.1 'Glasswork'** Thermal and bond energy, deducing equilibrium state, working with numbers
- **1.2 'Substance $\mathcal{A}$ in water'** Deducing heats, thermal equilibrium, sketching a three-phase plot
- **2.1 'On the slopes'** Kinetic and potential energy, calculating work, converting to heat
- **2.2 'Orbiting the sun'** Reading a potential, formulas and derivatives, sketching a potential
- **3.1 'Frozen modes'** Relating atomic structure to modes, particle model of thermal energy
- **3.2 'Bond energy in a cube'** Finding neighbors, particle model of bond energy
- **4.1 'Cycle of constants'** Work & Heat, heat capacities, sketching PV diagram, calculating entropy
- **4.2 'Fridge'** Work & Heat, sketching PV and TS diagrams, thinking of entropy

1 Applying Models to Thermal Phenomena

Question 1.1 'Glasswork'

A glassblower is working with a type of glass that melts at 1500 K,

Two-phase diagram of 1 kg of glass



The glassblower uses 0.5 kg of blue glass and 2.0 kg of red glass. With these, he does the following:

(i) Starting at room temperature (300 K), she separately heats the blue glass to 1500 K (without melting) and the red glass to 1800 K (so it is liquid).

(ii) She mixes the blue and red glass into one system and lets it reach thermal equilibrium.

(iii) She puts the mixed glass into a fridge to cool at a rate of 1 kJ/s.

Let us follow the procedure and understand the energies involved:

(a) Make new two-phase plots for the 0.5 kg of blue glass and the 2.0 kg of red glass, labeling the numbers on the axes.

(b) How much energy is added to the blue glass in step (i)? How much energy is added to the red glass?

(c) What is the temperature and phase of the glass once it reaches thermal equilibrium after mixing? What type(s) of energy does each glass gain/lose in step (ii)?

(d) How long does she need to wait for the glass to cool back down to room temperature in step (iii)?

(e) Repeat parts (b-d) for 0.25 kg of blue glass and for 1.0 kg of blue glass.

Question 1.2 'Substance $\mathcal{A}$ in water'

A group of researchers conduct a series of experiments with an unknown substance $\mathcal{A}$ to determine its three phase diagram. They have already determined that substance $\mathcal{A}$ has a melting point of 200K and a boiling point of 250K. With this information, they conduct the following experiment:

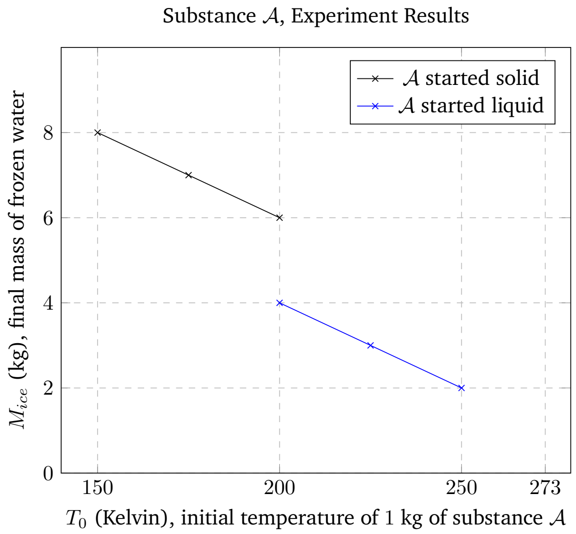
(i) Take 1 kg of substance $\mathcal{A}$ at a temperature T_0 in either the solid or liquid phase

(ii) Put the substance into a very large bath of liquid water that starts at a temperature of 273 K

(iii) Wait until the system reaches thermal equilibrium

(iv) Measure the weight of the water that froze M_{ice}

The following plot summarizes the results of their experiments:



Given the heat of melting for water ($\Delta H_{H_2O} = 333.5$ kJ/kg), use the three-phase model and the energy-interaction model to answer the following questions as you draw an temperature-energy plot for 1kg of the substance $\mathcal{A}$:

(a) What is the specific heat of solid $\mathcal{A}$?
Draw the line corresponding to heating $\mathcal{A}$ from 150 K to 200 K.

(b) What is the heat of melting of $\mathcal{A}$?
Draw the line corresponding to melting $\mathcal{A}$.

(c) What is the specific heat of liquid $\mathcal{A}$?
Draw the line corresponding to heating $\mathcal{A}$ from 200 K to 250 K.

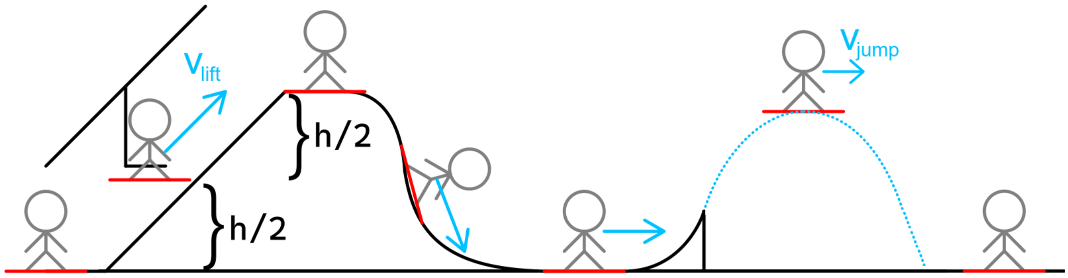
(d) How much energy does $\mathcal{A}$ absorb if it started as a liquid at $T_0 = 250$ K?

(e) Given that gaseous $\mathcal{A}$ has a (constant volume) specific heat of 11.12 kJ/(kg K), what is the heat of vaporization?
Draw the lines corresponding to vaporizing $\mathcal{A}$ and heating it to 0 K.

(f) Based on the ratio between the specific heat of solid $\mathcal{A}$ and gaseous $\mathcal{A}$, what can you say about the number of active modes that gaseous $\mathcal{A}$ has at 250 K?

2 Applying Models to Mechanical Phenomena

Question 2.1 'On the slopes'



A picture corresponding to the question. The velocity of the snowboarder is indicated by the light blue arrow. Not to scale.

The ski lift takes an initialy stationary snowboarder of mass M, moves them at a constant speed v_{lift}, and then releases them at the top of the hill of height h with zero speed. The snowboarder goes down the *frictionless* slope with no initial speed. He reaches a ramp, and makes a jump where he has a horizontal velocity of v_{jump}. The snowboarder lands on the other side of the ramp, and comes to a stop using friction with the snow.

(a) How much work does the lift do by the time the snowboarder is at a height $h/2$? How much more work does it take to get the snowboarder to h?
Hint: Remember to include the KE, and use $\Delta E = Q + W$.

(b) How much energy to the snowboarder have before he goes down the slope? At the bottom of the slope? At the peak of his jump?

(c) Calculate the speed when the snowboarder is moving fastest.

(d) Calculate the height of the snowboarder's jump.

(e) How much snow does the snowboarder melt as he comes to a stop after his jump?
Assume the snow is at $0°$C, and has a heat of melting of ΔH_{melt}, and that no energy is lost elsewhere.

Question 2.2 'Orbiting the sun'

The orbit of an object around the sun can be described by two forces. As a consequence of the sun's large mass, the object is pulled towards the sun by a gravitational force. As a consequence of the objects large speed and angular momentum, it is effectively repelled from the sun by a centrifugal force[a]. The corresponding potential has the form

$$PE(r) = \frac{A}{r^2} - \frac{B}{r}$$

where r is the distance between the object and the sun, and $A, B > 0$ are constants that depend on the mass and angular momentum.

(a) Which term corresponds to the gravitational force?

(b) What are the units of A and B? *Hint: Remember that PE is measured in units of energy (e.g. Joules), and the radius in units of distance (e.g. meters).*

Now, we will try drawing the potential.

(c) For what radius is the potential energy zero?

(d) What value does the potential energy approach for very, very large radii?

(e) What value does the potential energy approach for very, very small radii?

(f) For what radius is the force zero? *Hint: Requires derivative, use* $F(r) = -\frac{dV}{dr}$.

(g) What is the potential energy when the force is zero?

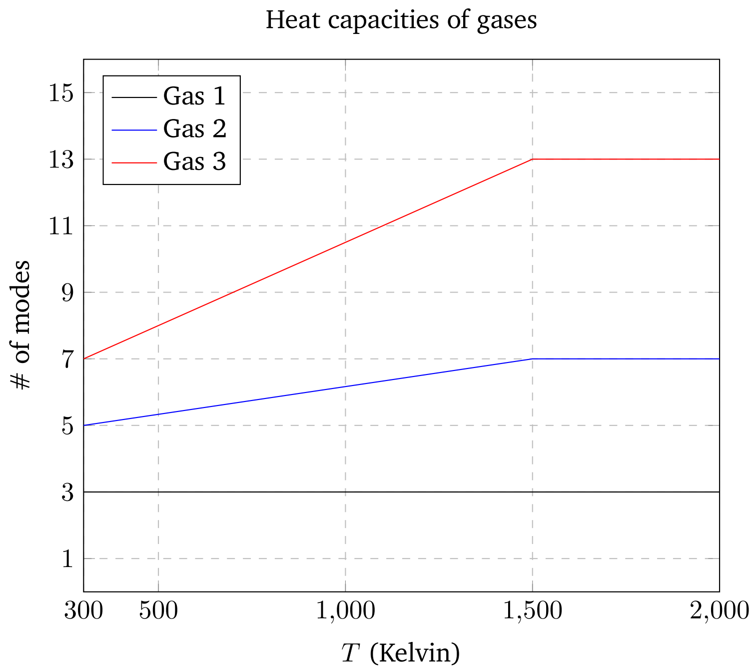
(h) Sketch the potential, and draw arrows corresponding to the force.

You may find it useful to compare this potential to the Lennard-Jones potential.

[a] This force is not a 'real' force like gravity, but rather an effect of trying to analyze the motion of an object as it rotates. You may learn more about the distinction in 7B, but for now you can use your skills to analyze the resulting potential.

3 Applying Particle Models to Matter

Question 3.1 'Frozen modes'



For the following questions, assume that all the rotational modes are already active at 300 K.

(a) Assuming none of the modes are frozen at $T = 2000$ K, how many atoms does each gas have?

(b) In one mole of particles of gas 2 at 500K, how many have 5 active modes?

(c) At what temperature (approximately) does every particle in gas 3 have 2 active vibrational modes?

(d) How much thermal energy per mole does each gas have at 2000 K? 500 K?

Using $E_{th} = \frac{nR}{2}(\text{\# of modes})T$, solve the following:

(e) Suppose 1 mole of gas 1 at 1500 K is mixed with a very large amount of gas 3 at 500 K. How much heat is exchanged between the two gases as they reach thermal equilibrium?

(f) Suppose 1 mole of gas 3 at 1500 K is mixed with a very large amount of gas 1 at 500 K. How much heat is exchanged between the two gases as they reach thermal equilibrium?

(g) Suppose 1 mole of gas 3 at 500 K is mixed with 1 mole of gas 2 at 1500 K. What temperature do they reach for thermal equilibrium?

Question 3.2 'Bond energy in a cube'

Consider a solid made out of atoms arranged in a cubic lattice. Let L denote the side-length of the cube, i.e. how many atoms long each side is. Let ε be the well depth for the Lennard-Jones potential corresponding to these bonds. First, let us think about nearest neighbors:

(a) How many atoms are there in a cube of size $L = 2$? Size $L = 3$? Arbitrary L?

(b) What value of L do you need to get approximately N_A atoms?

(c) How many nearest neighbor pairs are there in a cube of size $L = 2$? Size $L = 3$? Arbitrary L? *Hint: Requires being a bit creative. How many bonds are in one row of atoms? How many rows of atoms are there?*

(d) What is the average number of nearest neighbors? What does this value approach for very large L? Justify this value using the nearest neighbor approximation.

(e) What is the approximate bond energy in a cube of size L? What is the molar heat of melting such a cube?

Now, consider thinking of next-nearest neighbors. Letting r_0 be the distance to the nearest neighbors, use $LJ(\sqrt{2}r_0) = -0.24\varepsilon$ aand $LJ(\sqrt{3}r_0) = -0.07\varepsilon$.

(f) How many next-nearest neighbors do most atoms have? How far are they?

(g) How many next-next-nearest neighbors do most atoms have? How far are they?

(h) Recalculate the bond energy in a cube of size L using the next-nearest and next-next-nearest neighbors.

4 Models of Thermodynamics

Question 4.1 'Cycle of constants'

A mole of a monatomic ideal gas starts at $300K$ and has a volume of 1 m^3. The gas undergoes the following cycle:

(1) Raise temperature from 300 K to 600 K at constant pressure

(2) Lower temperature from 600 K to 300 K at constant volume

(3) Return to the original pressure and volume without changing the temperature

Using what you know about heat capacities and thermal energy, deduce the following:

(a) Sketch a PV diagram of the process, labeling the relevant values for each of the states.

(b) What is the work and heat corresponding to the constant pressure step?

(c) What is the work and heat corresponding to the constant volume step?

(d) What is the work for the constant temperature step? *Hint: Recall* $P = \frac{nRT}{V}$, $W = \int PdV$, *and* $\int_a^b \frac{1}{x}dx = \ln(b/a)$.

(e) What is the heat in the last step?

(f) What is the entropy in each step? For the whole cycle? *Hint: Use* $S = Q/\Delta T$ *for the isothermal step, and* $S = C\ln(T_f/T_i)$ *for the other steps. Remember the difference between* c_{vm} *and* c_{pm}.

Question 4.2 'Fridge'

A component of a refrigerator is a machine that throws heat away to the environment at the cost of having to perform some work. A simple model for the operation of this component consists of this cycle involving some steps with no heat (adiabatic) and some steps with no work:

(1) Refrigerant fluid expands adiabatically, lowering its temperature

(2) Cold refrigerant fluid enters the fridge and heats up without doing any work as it comes to a thermal equilibrium with the interior

(3) Refrigerant fluid contracts adiabatically, raising its temperature

(4) Hot refrigerant fluid leaves the fridge and cools down without doing any work as it comes into thermal equilibrium with the exterior

Now, answer these questions as you sketch the process on a PV and TS diagram.

(a) What is the sign on the work in steps 1 and 3?

(b) Explain why expanding adiabatically lowers temperature.

(c) What is the sign on ΔS in steps 2 and 4?

(d) What is the sign on ΔP in steps 2 and 4?

(e) Sketch a PV and a TS diagram for the cycle.

(f) Is the total heat positive or negative?

(g) How is the sign on the heat related to $\Delta S_{\text{closed}} \geq 0$? *Hint: Note that $\Delta S_{component} = 0$, since it goes trough a cycle. Think about ΔS_{room} and ΔS_{fridge}.*